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Topical REVISION NOTES

MATHEMATICS

Xander Yun MSc, PGDE, BSc

O LEVEL

Includes

✓ Detailed Worked Examples

✓ Comprehensive Revision Notes

✓ Effective Study Guide

Also suitable for **N(A)**

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MATHEMATICS

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PREFACE

O Level Mathematics Topical Revision Notes has been written in accordance with the latest syllabus issued by the Ministry of Education, Singapore.

This book is divided into 21 units, each covering a topic as laid out in the syllabus. Important concepts and formulae are highlighted in each unit, with relevant worked examples to help students learn how to apply theoretical knowledge to examination questions.

To make this book suitable for N(A) Level students, sections not applicable for the N(A) Level examination are indicated with a bar (▬).

We believe this book will be of great help to teachers teaching the subject and students preparing for their O Level and N(A) Level Mathematics examinations.

CONTENTS

UNIT 1.1 Numbers and the Four Operations

Numbers

1. The set of natural numbers, $\mathbb{N} = \{1, 2, 3, \ldots\}$

2. The set of whole numbers, $W = \{0, 1, 2, 3, \ldots\}$

3. The set of integers, $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

4. The set of positive integers, $\mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

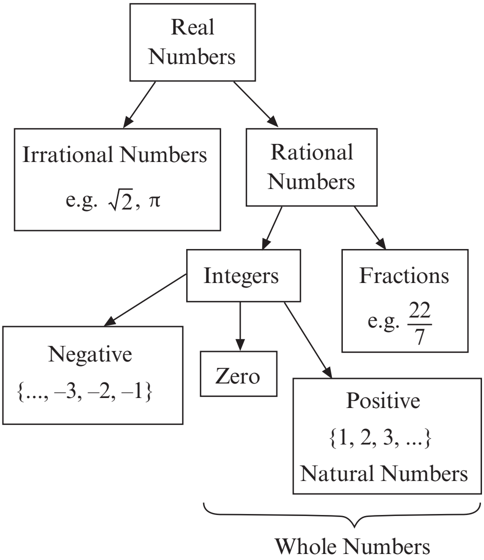
5. The set of negative integers, $\mathbb{Z}^- = \{-1, -2, -3, \ldots\}$

6. The set of rational numbers, $\mathbb{Q} = \{\frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0\}$

7. An irrational number is a number which cannot be expressed in the form $\frac{a}{b}$, where a, b are integers and $b \neq 0$.

8. The set of real numbers $\mathbb{R}$ is the set of rational and irrational numbers.

9.



Example 1

The temperature at the bottom of a mountain was 22 °C and the temperature at the top was –7 °C. Find

(a) the difference between the two temperatures,

(b) the average of the two temperatures.

Solution

(a) Difference between the temperatures = 22 – (–7)

= 22 + 7

= 29 °C

(b) Average of the temperatures $= \frac{22+(-7)}{2}$

$= \frac{22-7}{2}$

$= \frac{15}{2}$

= 7.5 °C

Prime Factorisation

10. A prime number is a number that can only be divided exactly by 1 and itself. However, 1 is not considered as a prime number.
e.g. 2, 3, 5, 7, 11, 13, …

11. Prime factorisation is the process of expressing a composite number as a product of its prime factors.

Example 2

Express 30 as a product of its prime factors.

Solution

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
Of these, 2, 3, 5 are prime factors.

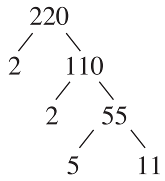
$$\begin{array}{r|l}
2 & 30 \\ \hline
3 & 15 \\ \hline
5 & 5 \\ \hline
 & 1
\end{array}$$

$\therefore 30 = 2 \times 3 \times 5$

Example 3

Express 220 as a product of its prime factors.

Solution



$\therefore 220 = 2^2 \times 5 \times 11$

Factors and Multiples

12. The highest common factor (HCF) of two or more numbers is the largest factor that is common to all the numbers.

Example 4

Find the highest common factor of 18 and 30.

Solution

$18 = 2 \times 3^2$
$30 = 2 \times 3 \times 5$

$$\begin{aligned} \text{HCF} &= 2 \times 3 \\ &= 6 \end{aligned}$$

Example 5

Find the highest common factor of 80, 120 and 280.

Solution

Method 1

$$\begin{array}{r|lll} 2 & 80, & 120, & 280 \\ \cline{2-4} 2 & 40, & 60, & 140 \\ \cline{2-4} 2 & 20, & 30, & 70 \\ \cline{2-4} 5 & 10, & 15, & 35 \\ \cline{2-4} \multicolumn{1}{r}{} & 2, & 3, & 7 \end{array}$$

(Since the three numbers cannot be divided further by a common prime factor, we stop here)

$$\begin{aligned} \text{HCF} &= 2 \times 2 \times 2 \times 5 \\ &= 40 \end{aligned}$$

Method 2

Express 80, 120 and 280 as products of their prime factors
$80 = 2^3 \times 5$
$120 = 2^3 \times 5$
$280 = 2^3 \times 5 \times 7$

$$\begin{aligned} \text{HCF} &= 2^3 \times 5 \\ &= 40 \end{aligned}$$

13. The lowest common multiple (LCM) of two or more numbers is the smallest multiple that is common to all the numbers.

Example 6

Find the lowest common multiple of 18 and 30.

Solution

$18 = 2 \times 3^2$
$30 = 2 \times 3 \times 5$

$\text{LCM} = 2 \times 3^2 \times 5$
$= 90$

Example 7

Find the lowest common multiple of 5, 15 and 30.

Solution

Method 1

$$\begin{array}{r|lll} 2 & 5, & 15, & 30 \\ \hline 3 & 5, & 15, & 15 \\ \hline 5 & 5, & 5, & 5 \\ \hline & 1, & 1, & 1 \end{array}$$

(Continue to divide by the prime factors until 1 is reached)

$\text{LCM} = 2 \times 3 \times 5$
$= 30$

Method 2

Express 5, 15 and 30 as products of their prime factors.
$5 = 1 \times 5$
$15 = 3 \times 5$
$30 = 2 \times 3 \times 5$

$\text{LCM} = 2 \times 3 \times 5$
$= 30$

Squares and Square Roots

14. A perfect square is a number whose square root is a whole number.

15. The square of a is a^2.

16. The square root of a is $\sqrt{a}$ or $a^{\frac{1}{2}}$.

Example 8

Find the square root of 256 without using a calculator.

Solution

$$\begin{array}{r|l}
2 & 256 \\ \cline{2-2}
2 & 128 \\ \cline{2-2}
2 & 64 \\ \cline{2-2}
2 & 32 \\ \cline{2-2}
2 & 16 \\ \cline{2-2}
2 & 8 \\ \cline{2-2}
2 & 4 \\ \cline{2-2}
2 & 2 \\ \cline{2-2}
\multicolumn{1}{r}{} & 1
\end{array}$$

(Continue to divide by the prime factors until 1 is reached)

$$\begin{aligned}
\sqrt{256} &= \sqrt{2^8} \\
&= 2^4 \\
&= 16
\end{aligned}$$

Example 9

Given that 30k is a perfect square, write down the value of the smallest integer k.

Solution

For $2 \times 3 \times 5 \times k$ to be a perfect square, the powers of its prime factors must be in multiples of 2,

i.e. $k = 2 \times 3 \times 5$

$= 30$

Cubes and Cube Roots

17. A perfect cube is a number whose cube root is a whole number.

18. The cube of a is a^3.

19. The cube root of a is $\sqrt[3]{a}$ or $a^{\frac{1}{3}}$.

Example 10

Find the cube root of 3375 without using a calculator.

Solution

$$\begin{array}{r|l}
3 & 3375 \\ \hline
3 & 1125 \\ \hline
3 & 375 \\ \hline
5 & 125 \\ \hline
5 & 25 \\ \hline
5 & 5 \\ \hline
 & 1
\end{array}$$

(Continue to divide by the prime factors until 1 is reached)

$$\begin{aligned}\sqrt[3]{3375} &= \sqrt[3]{3^3 \times 5^3} \\ &= 3 \times 5 \\ &= 15\end{aligned}$$

Reciprocal

20. The reciprocal of x is $\dfrac{1}{x}$.

21. The reciprocal of $\dfrac{x}{y}$ is $\dfrac{y}{x}$.

Significant Figures

22. All non-zero digits are significant.

23. A zero (or zeros) between non-zero digits is (are) significant.

24. In a whole number, zeros after the last non-zero digit may or may not be significant,
e.g. 7006 = 7000 (to 1 s.f.)
7006 = 7000 (to 2 s.f.)
7006 = 7010 (to 3 s.f.)
7436 = 7000 (to 1 s.f.)
7436 = 7400 (to 2 s.f.)
7436 = 7440 (to 3 s.f.)

Example 11

Express 2014 correct to
(a) 1 significant figure,
(b) 2 significant figures,
(c) 3 significant figures.

Solution

(a) $2014 = \underset{\substack{\uparrow \\ 1\text{ s.f.}}}{2}000$ (to 1 s.f.)

(b) $2014 = \underbrace{20}_{2\text{ s.f.}}00$ (to 2 s.f.)

(c) $2014 = \underbrace{201}_{3\text{ s.f.}}0$ (to 3 s.f.)

25. In a decimal, zeros before the first non-zero digit are not significant,
e.g. 0.006 09 = 0.006 (to 1 s.f.)
0.006 09 = 0.0061 (to 2 s.f.)
6.009 = 6.01 (to 3 s.f.)

26. In a decimal, zeros after the last non-zero digit are significant.

Example 12

(a) Express 2.0367 correct to 3 significant figures.
(b) Express 0.222 03 correct to 4 significant figures.

Solution

(a) 2.0367 = 2.04
(b) $0.222\,03 = 0.\underbrace{2220}_{4 \text{ s.f.}}$

Decimal Places

27. Include one extra figure for consideration. Simply drop the extra figure if it is less than 5. If it is 5 or more, add 1 to the previous figure before dropping the extra figure,
e.g. 0.7374 = 0.737 (to 3 d.p.)
5.0306 = 5.031 (to 3 d.p.)

Standard Form

28. Very large or small numbers are usually written in standard form $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer,
e.g. $1\,350\,000 = 1.35 \times 10^6$
$0.000\,875 = 8.75 \times 10^{-4}$

Example 13

The population of a country in 2012 was 4.05 million. In 2013, the population increased by 1.1×10^5. Find the population in 2013.

Solution

$$
\begin{aligned}
4.05 \text{ million} &= 4.05 \times 10^6 \\
\text{Population in 2013} &= 4.05 \times 10^6 + 1.1 \times 10^5 \\
&= 4.05 \times 10^6 + 0.11 \times 10^6 \\
&= (4.05 + 0.11) \times 10^6 \\
&= 4.16 \times 10^6
\end{aligned}
$$

Estimation

29. We can estimate the answer to a complex calculation by replacing numbers with approximate values for simpler calculation.

Example 14

Estimate the value of $\dfrac{3.49 \times \sqrt{35.7}}{35.1}$ correct to 1 significant figure.

Solution

$$
\begin{aligned}
\frac{3.49 \times \sqrt{35.7}}{35.1} &\approx \frac{3.5 \times \sqrt{36}}{35} \qquad \text{(Express each value to at least 2 s.f.)} \\
&= \frac{3.5}{35} \times \sqrt{36} \\
&= 0.1 \times 6 \\
&= 0.6 \text{ (to 1 s.f.)}
\end{aligned}
$$

Common Prefixes

30.

Power of 10	Name	SI Prefix	Symbol	Numerical Value
10^{12}	trillion	tera-	T	1 000 000 000 000
10^{9}	billion	giga-	G	1 000 000 000
10^{6}	million	mega-	M	1 000 000
10^{3}	thousand	kilo-	k	1000
10^{-3}	thousandth	milli-	m	$0.001 = \frac{1}{1000}$
10^{-6}	millionth	micro-	μ	$0.000\,001 = \frac{1}{1\,000\,000}$
10^{-9}	billionth	nano-	n	$0.000\,000\,001 = \frac{1}{1\,000\,000\,000}$
10^{-12}	trillionth	pico-	p	$0.000\,000\,000\,001 = \frac{1}{1\,000\,000\,000\,000}$

Example 15

Light rays travel at a speed of 3×10^8 m/s. The distance between Earth and the sun is 32 million km. Calculate the amount of time (in seconds) for light rays to reach Earth. Express your answer to the nearest minute.

Solution

$$\begin{aligned} 48 \text{ million km} &= 48 \times 1\,000\,000 \text{ km} \\ &= 48 \times 1\,000\,000 \times 1000 \text{ m} \quad (1 \text{ km} = 1000 \text{ m}) \\ &= 48\,000\,000\,000 \text{ m} \\ &= 48 \times 10^9 \text{ m} \\ &= 4.8 \times 10^{10} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Time} &= \frac{\text{Distance}}{\text{Speed}} \\ &= \frac{4.8 \times 10^9 \text{ m}}{3 \times 10^8 \text{ m/s}} \\ &= 16 \text{ s} \end{aligned}$$

Laws of Arithmetic

31. $a + b = b + a$ (Commutative Law)
$a \times b = b \times a$
$(p + q) + r = p + (q + r)$ (Associative Law)
$(p \times q) \times r = p \times (q \times r)$
$p \times (q + r) = p \times q + p \times r$ (Distributive Law)

32. When we have a few operations in an equation, take note of the order of operations as shown.
Step 1: Work out the expression in the **brackets** first. When there is more than 1 pair of brackets, work out the expression in the innermost brackets first.
Step 2: Calculate the **powers** and **roots**.
Step 3: **Divide** and **multiply** from left to right.
Step 4: **Add** and **subtract** from left to right.

Example 16

Calculate the value of the following.

(a) $2 + (5^2 - 4) \div 3$ **(b)** $14 - [45 - (26 + \sqrt{16})] \div 5$

Solution

(a)
$$\begin{aligned} 2 + (5^2 - 4) \div 3 &= 2 + (25 - 4) \div 3 && \text{(Power)} \\ &= 2 + 21 \div 3 && \text{(Brackets)} \\ &= 2 + 7 && \text{(Divide)} \\ &= 9 && \text{(Add)} \end{aligned}$$

(b)
$$\begin{aligned} 14 - [45 - (26 + \sqrt{16})] \div 5 &= 14 - [45 - (26 + 4)] \div 5 && \text{(Roots)} \\ &= 14 - [45 - 30] \div 5 && \text{(Innermost brackets)} \\ &= 14 - 15 \div 5 && \text{(Brackets)} \\ &= 14 - 3 && \text{(Divide)} \\ &= 11 && \text{(Subtract)} \end{aligned}$$

33. positive number × positive number = positive number
negative number × negative number = positive number
negative number × positive number = negative number
positive number ÷ positive number = positive number
negative number ÷ negative number = positive number
positive number ÷ negative number = negative number

Example 17

Simplify (–1) × 3 – (–3)(–2) ÷ (–2).

Solution

$$\begin{aligned}(-1)\times 3-(-3)(-2)\div(-2) &= -3-6\div(-2)\\ &= -3-(-3)\\ &= 0\end{aligned}$$

Laws of Indices

34. Law 1 of Indices: $a^m \times a^n = a^{m+n}$
Law 2 of Indices: $a^m \div a^n = a^{m-n}$, if $a \neq 0$
Law 3 of Indices: $(a^m)^n = a^{mn}$
Law 4 of Indices: $a^n \times b^n = (a \times b)^n$
Law 5 of Indices: $a^n \div b^n = \left(\frac{a}{b}\right)^n$, if $b \neq 0$

Example 18

(a) Given that $5^{18} \div 125 = 5^k$, find k.
(b) Simplify $3 \div 6p^{-4}$.

Solution

(a)
$$\begin{aligned}5^{18}\div 125 &= 5^k\\ 5^{18}\div 5^3 &= 5^k\\ 5^{18-3} &= 5^k\\ 5^{15} &= 5^k\\ \therefore k &= 15\end{aligned}$$

(b)
$$\begin{aligned}3\div 6p^{-4} &= \frac{3}{6p^{-4}}\\ &= \frac{p^4}{2}\end{aligned}$$

Zero Indices

35. If a is a real number and $a \neq 0, a^0 = 1$.

Negative Indices

36. If a is a real number and $a \neq 0, a^{-n} = \frac{1}{a^n}$.

Fractional Indices

37. If n is a positive integer, $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

38. If m and n are positive integers, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$, where $a > 0$.

Example 19

Simplify $\frac{3y^2}{5x} \div \frac{3x^2y}{20xy}$ and express your answer in positive indices.

Solution

$$
\begin{aligned}
\frac{3y^2}{5x} \div \frac{3x^2y}{20xy} &= \frac{3y^2}{5x} \times \frac{20xy}{3x^2y} \\
&= \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} y^2 x^{-1} \times \frac{\overset{4}{\cancel{20}}}{\underset{1}{\cancel{3}}} x^{-1} \\
&= 4x^{-2}y^2 \\
&= \frac{4y^2}{x^2}
\end{aligned}
$$

UNIT 1.2

Ratio, Rate and Proportion

Ratio

1. The ratio of a to b, written as $a : b$, is $a \div b$ or $\frac{a}{b}$, where $b \neq 0$ and $a, b \in \mathbb{Z}^+$.
2. A ratio has no units.

Example 1

In a stationery shop, the cost of a pen is \$1.50 and the cost of a pencil is 90 cents. Express the ratio of their prices in the simplest form.

Solution

We have to first convert the prices to the same units.

$$\$1.50 = 150 \text{ cents}$$

$$\begin{aligned}\text{Price of pen : Price of pencil} &= 150 : 90 \\ &= 5 : 3\end{aligned}$$

Map Scales

3. If the linear scale of a map is $1 : r$, it means that 1 cm on the map represents r cm on the actual piece of land.
4. If the linear scale of a map is $1 : r$, the corresponding area scale of the map is $1 : r^2$.

Example 2

In the map of a town, 10 km is represented by 5 cm.

(a) What is the actual distance if it is represented by a line of length 2 cm on the map?

(b) Express the map scale in the ratio 1 : n.

(c) Find the area of a plot of land that is represented by 10 cm^2.

Solution

(a) Given that the scale is 5 cm : 10 km

= 1 cm : 2 km

Therefore, 2 cm : 4 km

(b) Since 1 cm : 2 km,

$$\left.\begin{array}{l}1\text{ cm} : 2000\text{ m}\\ 1\text{ cm} : 200\,000\text{ cm}\end{array}\right\}\text{(Convert to the same units)}$$

Therefore, the map scale is 1 : 200 000.

(c) 1 cm : 2 km

1 cm^2 : 4 km^2

10 cm^2 : $10 \times 4 = 40\text{ km}^2$

Therefore, the area of the plot of land is 40 km^2.

Example 3

A length of 8 cm on a map represents an actual distance of 2 km. Find

(a) the actual distance represented by 25.6 cm on the map, giving your answer in km,

(b) the area on the map, in cm^2, which represents an actual area of 2.4 km^2,

(c) the scale of the map in the form 1 : *n*.

Solution

(a) 8 cm represent 2 km

1 cm represents $\frac{2}{8}$ km = 0.25 km

25.6 cm represents (0.25 × 25.6) km = 6.4 km

(b) 1 cm^2 represents (0.25^2) km^2 = 0.0625 km^2

0.0625 km^2 is represented by 1 cm^2

2.4 km^2 is represented by $\frac{2.4}{0.0625}$ cm^2 = 38.4 cm^2

(c) 1 cm represents 0.25 km = 25 000 cm

∴ Scale of map is 1 : 25 000

Direct Proportion

5. If y is directly proportional to x, then $y = kx$, where k is a constant and $k \neq 0$. Therefore, when the value of x increases, the value of y also increases proportionally by a constant k.

Example 4

Given that y is directly proportional to x, and $y = 5$ when $x = 10$, find y in terms of x.

Solution

Since y is directly proportional to x, we have $y = kx$.
When $x = 10$ and $y = 5$,

$$5 = k(10)$$
$$k = \frac{1}{2}$$

Hence, $y = \frac{1}{2}x$.

Example 5

2 m of wire costs \$10. Find the cost of a wire with a length of h m.

Solution

Let the length of the wire be x and the cost of the wire be y.

$$y = kx$$
$$10 = k(2)$$
$$k = 5$$

i.e. $y = 5x$

When $x = h$,

$$y = 5h$$

∴ The cost of a wire with a length of h m is \$5h.

Inverse Proportion

6. If y is inversely proportional to x, then $y = \frac{k}{x}$, where k is a constant and $k \neq 0$.

Example 6

Given that y is inversely proportional to x, and $y = 5$ when $x = 10$, find y in terms of x.

Solution

Since y is inversely proportional to x, we have

$$y = \frac{k}{x}$$

When $x = 10$ and $y = 5$,

$$5 = \frac{k}{10}$$

$$k = 50$$

Hence, $y = \dfrac{50}{x}$.

Example 7

7 men can dig a trench in 5 hours. How long will it take 3 men to dig the same trench?

Solution

Let the number of men be x and the number of hours be y.

$$y = \frac{k}{x}$$

$$5 = \frac{k}{7}$$

$$k = 35$$

i.e. $y = \dfrac{35}{x}$

When $x = 3$,

$$\begin{aligned} y &= \frac{35}{3} \\ &= 11\frac{2}{3} \end{aligned}$$

$\therefore$ It will take $11\frac{2}{3}$ hours.

Equivalent Ratio

7. To attain equivalent ratios involving fractions, we have to multiply or divide the numbers of the ratio by the LCM.

Example 8

$\frac{1}{4}$ cup of sugar, $1\frac{1}{2}$ cup of flour and $\frac{5}{6}$ cup of water are needed to make a cake. Express the ratio using whole numbers.

Solution

$$\begin{array}{cccccl} \text{Sugar} & : & \text{Flour} & : & \text{Water} & \\ \frac{1}{4} & : & 1\frac{1}{2} & : & \frac{5}{6} & \\ \frac{1}{4} \times 12 & : & 1\frac{1}{2} \times 12 & : & \frac{5}{6} \times 12 & \text{(Multiply throughout by the LCM, which is 12)} \\ 3 & : & 18 & : & 10 & \end{array}$$

UNIT 1.3 Percentage

Percentage

1. A percentage is a fraction with denominator 100,

i.e. $x\%$ means $\frac{x}{100}$.

2. To convert a fraction to a percentage, multiply the fraction by 100%,

e.g. $\frac{3}{4} \times 100\% = 75\%$.

3. To convert a percentage to a fraction, divide the percentage by 100%,

e.g. $75\% = \frac{75}{100} = \frac{3}{4}$.

4. New value = Final percentage × Original value

5. Increase (or decrease) = Percentage increase (or decrease) × Original value

6. $\text{Percentage increase} = \frac{\text{Increase in quantity}}{\text{Original quantity}} \times 100\%$

$\text{Percentage decrease} = \frac{\text{Decrease in quantity}}{\text{Original quantity}} \times 100\%$

Example 1

A car petrol tank which can hold 60 l of petrol is spoilt and it leaks about 5% of petrol every 8 hours. What is the volume of petrol left in the tank after a whole full day?

Solution

There are 24 hours in a full day.
After the first 8 hours,

$$\text{Amount of petrol left} = \frac{95}{100} \times 60$$
$$= 57\ l$$

After the next 8 hours,

$$\text{Amount of petrol left} = \frac{95}{100} \times 57$$
$$= 54.15\ l$$

After the last 8 hours,

$$\text{Amount of petrol left} = \frac{95}{100} \times 54.15$$
$$= 51.4\ l \text{ (to 3 s.f.)}$$

Example 2

Mr Wong is a salesman. He is paid a basic salary and a year-end bonus of 1.5% of the value of the sales that he had made during the year.

(a) In 2011, his basic salary was $2550 per month and the value of his sales was $234 000. Calculate the total income that he received in 2011.

(b) His basic salary in 2011 was an increase of 2% of his basic salary in 2010. Find his annual basic salary in 2010.

(c) In 2012, his total basic salary was increased to $33 600 and his total income was $39 870.

  (i) Calculate the percentage increase in his basic salary from 2011 to 2012.

  (ii) Find the value of the sales that he made in 2012.

(d) In 2013, his basic salary was unchanged as $33 600 but the percentage used to calculate his bonus was changed. The value of his sales was $256 000 and his total income was $38 720. Find the percentage used to calculate his bonus in 2013.

Solution

(a) Annual basic salary in 2011 = \$2550 × 12

= \$30 600

Bonus in 2011 = $\frac{1.5}{100} \times \$234\ 000$

= \$3510

Total income in 2011 = \$30 600 + \$3510

= \$34 110

(b) Annual basic salary in 2010 = $\frac{100}{102} \times \$2550 \times 12$

= \$30 000

(c) **(i)** Percentage increase = $\frac{\$33\ 600 - \$30\ 600}{\$30\ 600} \times 100\%$

= 9.80% (to 3 s.f.)

(ii) Bonus in 2012 = \$39 870 – \$33 600

= \$6270

Sales made in 2012 = $\frac{\$6270}{1.5} \times 100$

= \$418 000

(d) Bonus in 2013 = \$38 720 – \$33 600

= \$5120

Percentage used = $\frac{\$5120}{\$256\ 000} \times 100$

= 2%

UNIT 1.4 Speed

Speed

1. Speed is defined as the amount of distance travelled per unit time.

$$\text{Speed} = \frac{\text{Distance Travelled}}{\text{Time}}$$

Constant Speed

2. If the speed of an object does not change throughout the journey, it is said to be travelling at a constant speed.

Example 1

A bike travels at a constant speed of 10.0 m/s. It takes 2000 s to travel from Jurong to East Coast. Determine the distance between the two locations.

Solution

Speed: $v = 10$ m/s
Time: $t = 2000$ s
Distance: $d = vt$
$= 10 \times 2000$
$= 20\ 000$ m or 20 km

Average Speed

3. To calculate the average speed of an object, use the formula

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}.$$

Example 2

Tom travelled 105 km in 2.5 hours before stopping for lunch for half an hour. He then continued another 55 km for an hour. What was the average speed of his journey in km/h?

Solution

$$\text{Average speed} = \frac{105+55}{(2.5 + 0.5 + 1)}$$
$$= 40 \text{ km/h}$$

Example 3

Calculate the average speed of a spider which travels 250 m in $1\frac{1}{2}$ minutes. Give your answer in metres per second.

Solution

$1\frac{1}{2}$ min = 90 s

$$\text{Average speed} = \frac{250}{90}$$
$$= 2.78 \text{ m/s (to 3 s.f.)}$$

Conversion of Units

4. Distance:

1 m = 100 cm
1 km = 1000 m

5. Time:

1 h = 60 min
1 min = 60 s

6. Speed:

1 m/s = 3.6 km/h

Example 4

Convert 100 km/h to m/s.

Solution

$$100 \text{ km/h} = \frac{100\ 000}{3600} \text{ m/s} \quad (1 \text{ km} = 1000 \text{ m})$$
$$= 27.8 \text{ m/s (to 3 s.f.)}$$

7. Area:

$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$
$1 \text{ km}^2 = 1\ 000\ 000 \text{ m}^2$
$1 \text{ hectare} = 10\ 000 \text{ m}^2$

Example 5

Convert 2 hectares to cm^2.

Solution

$$2 \text{ hectares} = 2 \times 10\ 000 \text{ m}^2 \quad (\text{Convert to m}^2)$$
$$= 20\ 000 \times 10\ 000 \text{ cm}^2 \quad (\text{Convert to cm}^2)$$
$$= 200\ 000\ 000 \text{ cm}^2$$

8. Volume:

$1 \text{ m}^3 = 1\ 000\ 000 \text{ cm}^3$
$1\ l = 1000 \text{ m}l$
$= 1000 \text{ cm}^3$

Example 6

Convert 2000 cm^3 to m^3.

Solution

Since 1 000 000 cm^3 = 1 m^3,

$$2000\ \text{cm}^3 = \frac{2000}{1\ 000\ 000}$$
$$= 0.002\ \text{cm}^3$$

---

9. Mass:

1 kg = 1000 g

1 g = 1000 mg

1 tonne = 1000 kg

Example 7

Convert 50 mg to kg.

Solution

Since 1000 mg = 1 g,

$$50\ \text{mg} = \frac{50}{1000}\ \text{g} \quad \text{(Convert to g first)}$$
$$= 0.05\ \text{g}$$

Since 1000 g = 1 kg,

$$0.05\ \text{g} = \frac{0.05}{1000}\ \text{kg}$$
$$= 0.000\ 05\ \text{kg}$$

UNIT 1.5

Algebraic Representation and Formulae

Number Patterns

1. A number pattern is a sequence of numbers that follows an observable pattern.

e.g. 1st term, 2nd term, 3rd term, 4th term

1 , 3 , 5 , 7 , …

nth term denotes the general term for the number pattern.

2. Number patterns may have a common difference.

e.g. This is a sequence of even numbers.

$$\overset{+2}{\curvearrowright}\ \overset{+2}{\curvearrowright}\ \overset{+2}{\curvearrowright}$$
$$2,\ 4,\ 6,\ 8\ \ldots$$

This is a sequence of odd numbers.

$$\overset{+2}{\curvearrowright}\ \overset{+2}{\curvearrowright}\ \overset{+2}{\curvearrowright}$$
$$1,\ 3,\ 5,\ 7\ \ldots$$

This is a decreasing sequence with a common difference.

$$\overset{-3}{\curvearrowright}\ \overset{-3}{\curvearrowright}\ \overset{-3}{\curvearrowright}\ \overset{-3}{\curvearrowright}$$
$$19,\ 16,\ 13,\ 10,\ 7\ \ldots$$

3. Number patterns may have a common ratio.

e.g. This is a sequence with a common ratio.

$$\overset{\times 2}{\curvearrowright}\ \overset{\times 2}{\curvearrowright}\ \overset{\times 2}{\curvearrowright}\ \overset{\times 2}{\curvearrowright}$$
$$1,\ 2,\ 4,\ 8,\ 16\ \ldots$$

$$\overset{\div 2}{\curvearrowright}\ \overset{\div 2}{\curvearrowright}\ \overset{\div 2}{\curvearrowright}$$
$$128,\ 64,\ 32,\ 16\ \ldots$$

4. Number patterns may be perfect squares or perfect cubes.

e.g. This is a sequence of perfect squares.

$$1^2\ \ 2^2\ \ 3^2\ \ 4^2\ \ 5^2$$
$$1,\ 4,\ 9,\ 16,\ 25\ \ldots$$

This is a sequence of perfect cubes.

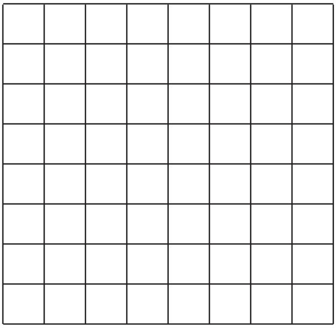
$$6^3\ \ 5^3\ \ 4^3\ \ 3^3\ \ 2^3$$
$$216,\ 125,\ 64,\ 27,\ 8\ \ldots$$

Example 1

How many squares are there on a 8 × 8 chess board?

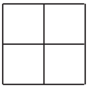
Solution

A chess board is made up of 8 × 8 squares.

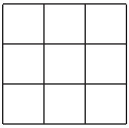


We can solve the problem by reducing it to a simpler problem.

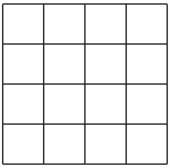
There is 1 square in a 1 × 1 square.



There are 4 + 1 squares in a 2 × 2 square.



There are 9 + 4 + 1 squares in a 3 × 3 square.



There are 16 + 9 + 4 + 1 squares in a 4 × 4 square.

Study the pattern in the table.

Size of square	Number of squares
1×1	$1 = 1^2$
2×2	$4 + 1 = 2^2 + 1^2$
3×3	$9 + 4 + 1 = 3^2 + 2^2 + 1^2$
4×4	$16 + 9 + 4 + 1 = 4^2 + 3^2 + 2^2 + 1^2$
$\vdots$	$\vdots$
8×8	$8^2 + 7^2 + 6^2 + \ldots + 1^2$

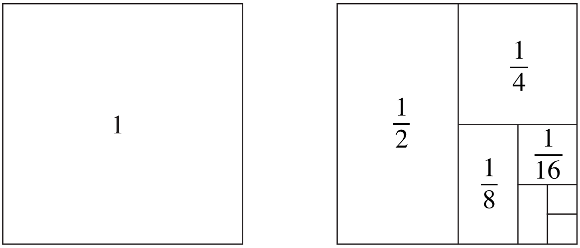
$\therefore$ The chess board has $8^2 + 7^2 + 6^2 + \ldots + 1^2 = 204$ squares.

Example 2

Find the value of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$.

Solution

We can solve this problem by drawing a diagram.
Draw a square of side 1 unit and let its area, i.e. 1 unit^2, represent the first number in the pattern. Do the same for the rest of the numbers in the pattern by drawing another square and dividing it into the fractions accordingly.



From the diagram, we can see that $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ is the total area of the two squares, i.e. 2 units^2.

$\therefore\ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots = 2$

5. Number patterns may have a combination of common difference and common ratio.
e.g. This sequence involves both a common difference and a common ratio.

$+3 \quad \times 2 \quad +3 \quad \times 2 \quad +3$

2, 5, 10, 13, 26, 29 ...

6. Number patterns may involve other sequences.
e.g. This number pattern involves the Fibonacci sequence.

$0+1 \quad 1+1 \quad 1+2 \quad 2+3 \quad 3+5 \quad 5+8 \quad 8+13$

0 , 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , 34 , . . .

Example 3

The first five terms of a number pattern are 4, 7, 10, 13 and 16.

(a) What is the next term?

(b) Write down the *n*th term of the sequence.

Solution

(a) 19

(b) $3n + 1$

Example 4

(a) Write down the 4th term in the sequence
2, 5, 10, 17, …

(b) Write down an expression, in terms of n, for the n^{th} term in the sequence.

Solution

(a) 4th term = 26

(b) n^{th} term $= n^2 + 1$

Basic Rules on Algebraic Expression

7.
- $kx = k \times x$ (Where k is a constant)
- $3x = 3 \times x$
  $= x + x + x$
- $x^2 = x \times x$
- $kx^2 = k \times x \times x$
- $x^2y = x \times x \times y$
- $(kx)^2 = kx \times kx$

8.
- $\dfrac{x}{y} = x \div y$
- $\dfrac{2 \pm x}{3} = (2 \pm x) \div 3$
  $= (2 \pm x) \times \dfrac{1}{3}$

Example 5

A cuboid has dimensions l cm by b cm by h cm. Find
(i) an expression for V, the volume of the cuboid,
(ii) the value of V when $l = 5$, $b = 2$ and $h = 10$.

Solution

(i) $V = l \times b \times h = lbh$
(ii) When $l = 5$, $b = 2$ and $h = 10$,

$$\begin{aligned} V &= (5)(2)(10) \\ &= 100 \end{aligned}$$

Example 6

Simplify $(y \times y + 3 \times y) \div 3$.

Solution

$$\begin{aligned} (y \times y + 3 \times y) \div 3 &= (y^2 + 3y) \div 3 \\ &= \frac{y^2 + 3y}{3} \end{aligned}$$

UNIT 1.6 Algebraic Manipulation

Expansion

1. $(a + b)^2 = a^2 + 2ab + b^2$

2. $(a - b)^2 = a^2 - 2ab + b^2$

3. $(a + b)(a - b) = a^2 - b^2$

Example 1

Expand $(2a - 3b^2 + 2)(a + b)$.

Solution

$$\begin{aligned}(2a - 3b^2 + 2)(a + b) &= (2a^2 - 3ab^2 + 2a) + (2ab - 3b^3 + 2b) \\ &= 2a^2 - 3ab^2 + 2a + 2ab - 3b^3 + 2b\end{aligned}$$

Example 2

Simplify $-8(3a - 7) + 5(2a + 3)$.

Solution

$$\begin{aligned}-8(3a - 7) + 5(2a + 3) &= -24a + 56 + 10a + 15 \\ &= -14a + 71\end{aligned}$$

Example 3

Solve each of the following equations.

(a) $2(x - 3) + 5(x - 2) = 19$

(b) $\frac{2y+6}{9} - \frac{5y}{12} = 3$

Solution

(a)
$$\begin{aligned} 2(x - 3) + 5(x - 2) &= 19 \\ 2x - 6 + 5x - 10 &= 19 \\ 7x - 16 &= 19 \\ 7x &= 35 \\ x &= 5 \end{aligned}$$

(b)
$$\begin{aligned} \frac{2y+6}{9} - \frac{5y}{12} &= 3 \\ \frac{4(2y+6)-3(5y)}{36} &= 3 \\ \frac{8y+24-15y}{36} &= 3 \\ \frac{-7y+24}{36} &= 3 \\ -7y + 24 &= 3(36) \\ 7y &= -84 \\ y &= -12 \end{aligned}$$

Factorisation

4. An algebraic expression may be factorised by extracting common factors,
e.g. $6a^3b - 2a^2b + 8ab = 2ab(3a^2 - a + 4)$

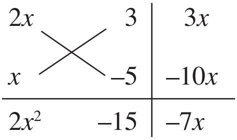
5. An algebraic expression may be factorised by grouping,
e.g.
$$\begin{aligned} 6a + 15ab - 10b - 9a^2 &= 6a - 9a^2 + 15ab - 10b \\ &= 3a(2 - 3a) + 5b(3a - 2) \\ &= 3a(2 - 3a) - 5b(2 - 3a) \\ &= (2 - 3a)(3a - 5b) \end{aligned}$$

6. An algebraic expression may be factorised by using the formula $a^2 - b^2 = (a + b)(a - b)$,

e.g. $$\begin{aligned} 81p^4 - 16 &= (9p^2)^2 - 4^2 \\ &= (9p^2 + 4)(9p^2 - 4) \\ &= (9p^2 + 4)(3p + 2)(3p - 2) \end{aligned}$$

7. An algebraic expression may be factorised by inspection,

e.g. $2x^2 - 7x - 15 = (2x + 3)(x - 5)$



Example 4

Solve the equation $3x^2 - 2x - 8 = 0$.

Solution

$$3x^2 - 2x - 8 = 0$$
$$(x - 2)(3x + 4) = 0$$

$x - 2 = 0$ or $3x + 4 = 0$

$x = 2$ $\qquad$ $x = -\frac{4}{3}$

Example 5

Solve the equation $2x^2 + 5x - 12 = 0$.

Solution

$$2x^2 + 5x - 12 = 0$$
$$(2x - 3)(x + 4) = 0$$

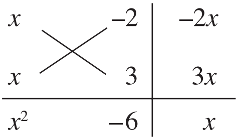
$$2x - 3 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = \frac{3}{2} \qquad\qquad x = -4$$

Example 6

Solve $2x + x = \dfrac{12 + x}{x}$.

Solution

$$2x + 3 = \frac{12 + x}{x}$$
$$2x^2 + 3x = 12 + x$$
$$2x^2 + 2x - 12 = 0$$
$$x^2 + x - 6 = 0$$
$$(x - 2)(x + 3) = 0$$
$$\therefore\ x = 2 \text{ or } x = -3$$



Addition and Subtraction of Fractions

8. To add or subtract algebraic fractions, we have to convert all the denominators to a common denominator.

Example 7

Express each of the following as a single fraction.

(a) $\frac{x}{3} + \frac{y}{5}$

(b) $\frac{3}{ab^3} + \frac{5}{a^2b}$

(c) $\frac{3}{x-y} + \frac{5}{y-x}$

(d) $\frac{6}{x^2-9} + \frac{3}{x-3}$

Solution

(a)
$$\begin{aligned}\frac{x}{3} + \frac{y}{5} &= \frac{5x}{15} + \frac{3y}{15} \qquad \text{(Common denominator = 15)}\\ &= \frac{5x+3y}{15}\end{aligned}$$

(b)
$$\begin{aligned}\frac{3}{ab^3} + \frac{5}{a^2b} &= \frac{3a}{a^2b^3} + \frac{5b^2}{a^2b^3} \qquad \text{(Common denominator = } a^2b^3\text{)}\\ &= \frac{3a+5b^2}{a^2b^3}\end{aligned}$$

(c)
$$\begin{aligned}\frac{3}{x-y} + \frac{5}{y-x} &= \frac{3}{x-y} - \frac{5}{x-y} \qquad \text{(Common denominator = } x-y\text{)}\\ &= \frac{3-5}{x-y}\\ &= \frac{-2}{x-y}\\ &= \frac{2}{y-x}\end{aligned}$$

(d)
$$\begin{aligned}\frac{6}{x^2-9} + \frac{3}{x-3} &= \frac{6}{(x+3)(x-3)} + \frac{3}{x-3}\\ &= \frac{6+3(x+3)}{(x+3)(x-3)} \qquad \text{(Common denominator = } (x+3)(x-3)\text{)}\\ &= \frac{6+3x+9}{(x+3)(x-3)}\\ &= \frac{3x+15}{(x+3)(x-3)}\end{aligned}$$

Example 8

Solve $\frac{4}{3b-6} + \frac{5}{4b-8} = 2.$

Solution

$$\frac{4}{3b-6} + \frac{5}{4b-8} = 2 \qquad \text{(Common denominator = 12(}b-2\text{))}$$

$$\frac{4}{3(b-2)} + \frac{5}{4(b-2)} = 2$$

$$\frac{4(4)+5(3)}{12(b-2)} = 2$$

$$\frac{31}{12(b-2)} = 2$$

$$31 = 24(b-2)$$

$$24b = 31 + 48$$

$$b = \frac{79}{24}$$

$$= 3\frac{7}{24}$$

Multiplication and Division of Fractions

9. To multiply algebraic fractions, we have to factorise the expression before cancelling the common terms. To divide algebraic fractions, we have to invert the divisor and change the sign from ÷ to ×.

Example 9

Simplify.

(a) $\frac{x+y}{3x-3y} \times \frac{2x-2y}{5x+5y}$

(b) $\frac{6p^3}{7qr} \div \frac{12p}{21q^2}$

(c) $\frac{x+y}{2x-y} \div \frac{2x+2y}{4x-2y}$

Solution

(a) $$\frac{x+y}{3x-3y} \times \frac{2x-2y}{5x+5y} = \frac{x+y}{3(x-y)} \times \frac{2(x-y)}{5(x+y)}$$
$$= \frac{1}{3} \times \frac{2}{5}$$
$$= \frac{2}{15}$$

(b) $$\frac{6p^3}{7qr} \div \frac{12p}{21q^2} = \frac{6p^3}{7qr} \times \frac{21q^2}{12p}$$
$$= \frac{3p^2q}{2r}$$

(c) $$\frac{x+y}{2x-y} \div \frac{2x+2y}{4x-2y} = \frac{x+y}{2x-y} \times \frac{4x-2y}{2x+2y}$$
$$= \frac{x+y}{2x-y} \times \frac{2(2x-y)}{2(x+y)}$$
$$= 1$$

Changing the Subject of a Formula

10. The subject of a formula is the variable which is written explicitly in terms of other given variables.

Example 10

Make t the subject of the formula, $v = u + at$.

Solution

To make t the subject of the formula,

$$v - u = at$$
$$t = \frac{v-u}{a}$$

Example 11

Given that the volume of the sphere is $V = \frac{4}{3}\pi r^3$, express r in terms of V.

Solution

$$V = \frac{4}{3}\pi r^3$$

$$r^3 = \frac{3}{4\pi}V$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

Example 12

Given that $T = 2\pi\sqrt{\frac{L}{g}}$, express L in terms of π, T and g.

Solution

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

$$\frac{T^2}{4\pi^2} = \frac{L}{g}$$

$$L = \frac{gT^2}{4\pi^2}$$

UNIT 1.7 Functions and Graphs

Linear Graphs

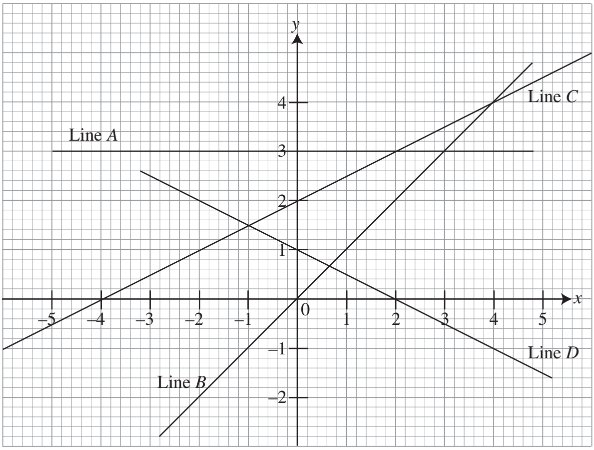
1. The equation of a straight line is given by $y = mx + c$, where m = gradient of the straight line and c = y-intercept.

2. The gradient of the line (usually represented by m) is given as

$$m = \frac{\text{vertical change}}{\text{horizontal change}} \text{ or } \frac{\text{rise}}{\text{run}}.$$

Example 1

Find the m (gradient), c (y-intercept) and equations of the following lines.



For Line A:

The line cuts the y-axis at $y = 3$. $\therefore$ $c = 3$.

Since Line A is a horizontal line, the vertical change = 0. $\therefore$ $m = 0$.

For Line *B*:

The line cuts the y-axis at $y = 0$. $\therefore\ c = 0$.

Vertical change = 1, horizontal change = 1.

$$\therefore\ m = \frac{1}{1} = 1$$

For Line *C*:

The line cuts the y-axis at $y = 2$. $\therefore\ c = 2$.

Vertical change = 2, horizontal change = 4.

$$\therefore\ m = \frac{2}{4} = \frac{1}{2}$$

For Line *D*:

The line cuts the y-axis at $y = 1$. $\therefore\ c = 1$.

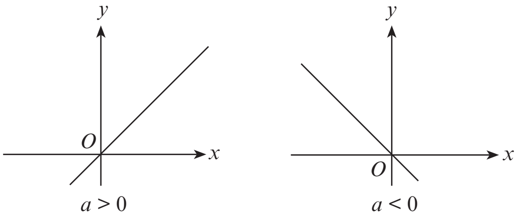
Vertical change = 1, horizontal change = –2.

$$\therefore\ m = \frac{1}{-2} = -\frac{1}{2}$$

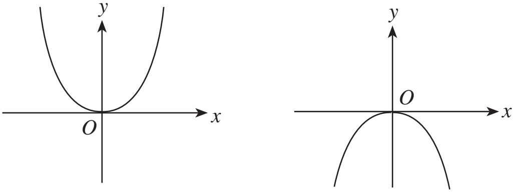
Line	$\boldsymbol{m}$	$\boldsymbol{c}$	**Equation**
A	0	3	$y = 3$
B	1	0	$y = x$
C	$\frac{1}{2}$	2	$y = \frac{1}{2}x + 2$
D	$-\frac{1}{2}$	1	$y = -\frac{1}{2}x + 1$

Graphs of $y = ax^n$

3. Graphs of $y = ax$

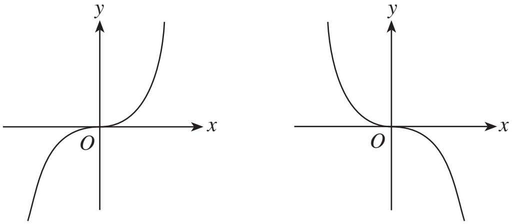


4. Graphs of $y = ax^2$



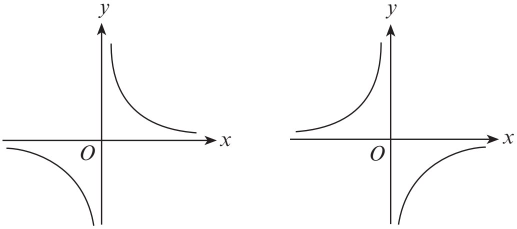
$a > 0$ $a < 0$

5. Graphs of $y = ax^3$



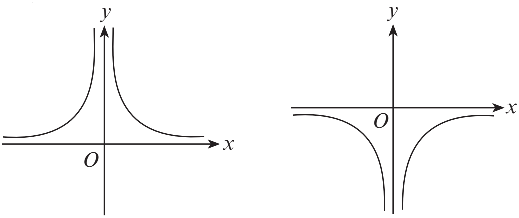
$a > 0$ $a < 0$

6. Graphs of $y = \dfrac{a}{x}$



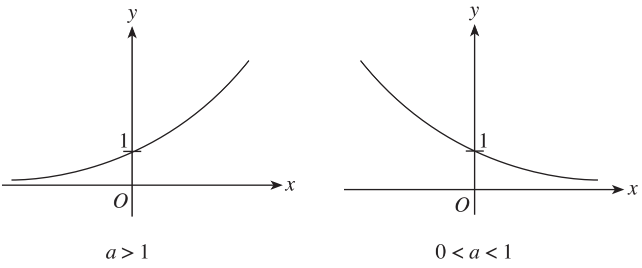
$a > 0$ $a < 0$

7. Graphs of $y = \dfrac{a}{x^2}$



$a > 0$ $a < 0$

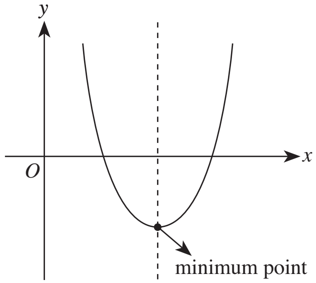
8. Graphs of $y = a^x$



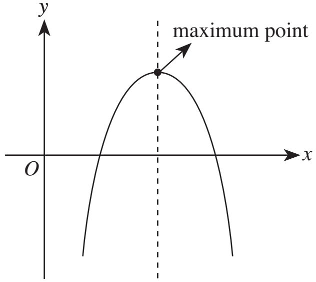
Graphs of Quadratic Functions

9. A graph of a quadratic function may be in the forms $y = ax^2 + bx + c$, $y = \pm(x - p)^2 + q$ and $y = \pm(x - h)(x - k)$.

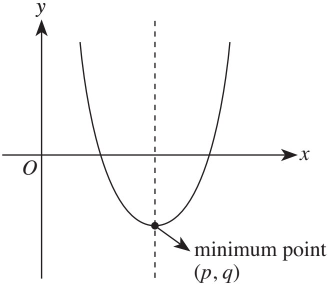
10. If $a > 0$, the quadratic graph has a minimum point.



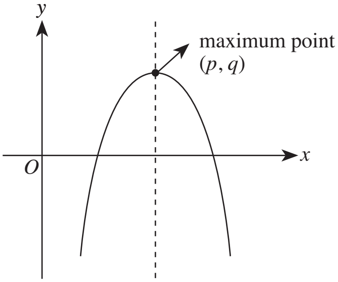
11. If $a < 0$, the quadratic graph has a maximum point.



12. If the quadratic function is in the form $y = (x - p)^2 + q$, it has a minimum point at (p, q).



13. If the quadratic function is in the form $y = -(x - p)^2 + q$, it has a maximum point at (p, q).



14. To find the x-intercepts, let $y = 0$.
To find the y-intercept, let $x = 0$.

15. To find the gradient of the graph at a given point, draw a tangent at the point and calculate its gradient.

16. The line of symmetry of the graph in the form $y = \pm(x - p)^2 + q$ is $x = p$.

17. The line of symmetry of the graph in the form $y = \pm(x - h)(x - k)$ is $x = \dfrac{h+k}{2}$.

Graph Sketching

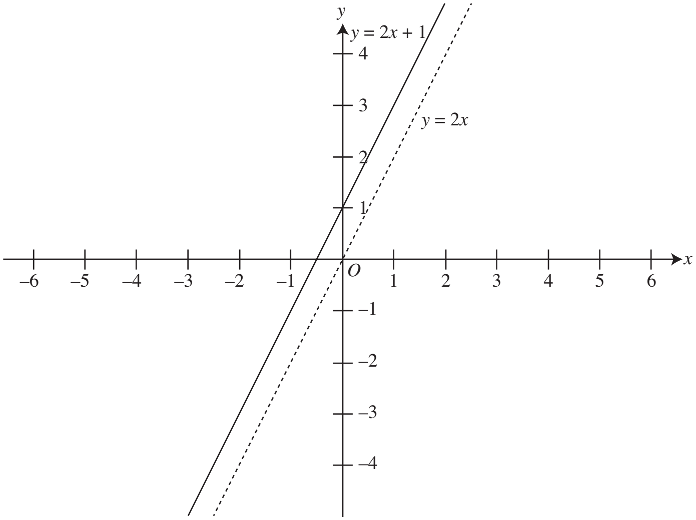
18. To sketch linear graphs with equations such as $y = mx + c$, shift the graph $y = mx$ upwards by c units.

Example 2

Sketch the graph of $y = 2x + 1$.

Solution

First, sketch the graph of $y = 2x$.
Next, shift the graph upwards by 1 unit.



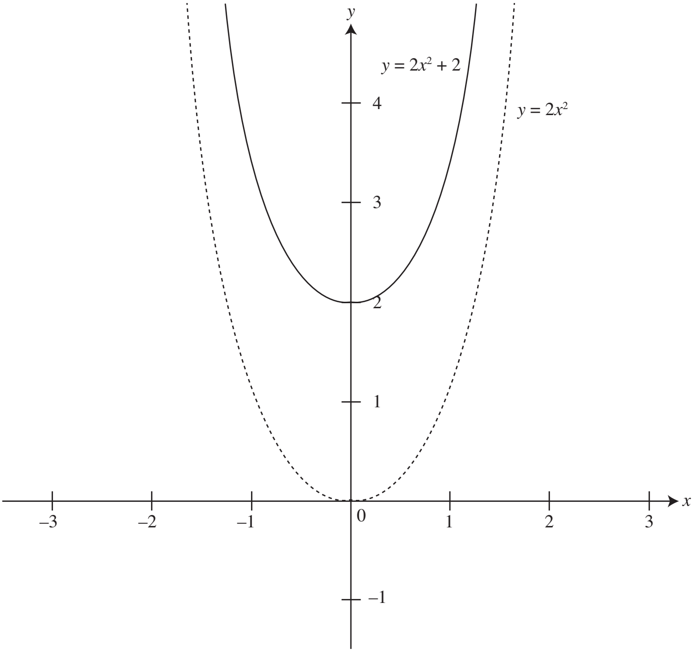
19. To sketch $y = ax^2 + b$, shift the graph $y = ax^2$ upwards by b units.

Example 3

Sketch the graph of $y = 2x^2 + 2$.

Solution

First, sketch the graph of $y = 2x^2$.
Next, shift the graph upwards by 2 units.



Graphical Solution of Equations

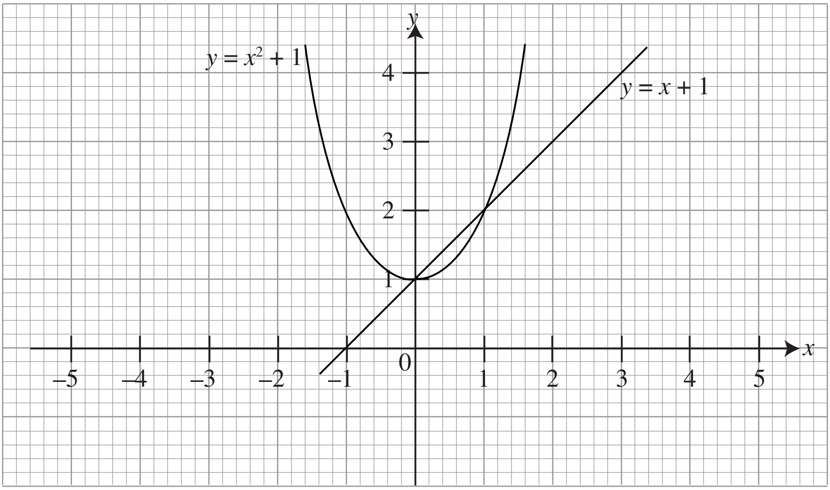
20. Simultaneous equations can be solved by drawing the graphs of the equations and reading the coordinates of the point(s) of intersection.

Example 4

Draw the graph of $y = x^2 + 1$. Hence, find the solutions to $x^2 + 1 = x + 1$.

Solution

x	–2	–1	0	1	2
y	5	2	1	2	5



Plot the straight line graph $y = x + 1$ in the axes above,
The line intersects the curve at $x = 0$ and $x = 1$.
When $x = 0$, $y = 1$.
When $x = 1$, $y = 2$.

$\therefore$ The solutions are (0, 1) and (1, 2).

Example 5

The table below gives some values of x and the corresponding values of y, correct to two decimal places, where $y = x(2 + x)(3 - x)$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	0	-3.38	-4	-2.63	0	3.13	p	7.88	8	5.63	q

(a) Find the value of p and of q.

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \leqslant x \leqslant 4$. Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-4 \leqslant y \leqslant 10$. On your axes, plot the points given in the table and join them with a smooth curve.

(c) Using your graph, find the values of x for which $y = 3$.

(d) By drawing a tangent, find the gradient of the curve at the point where $x = 2$.

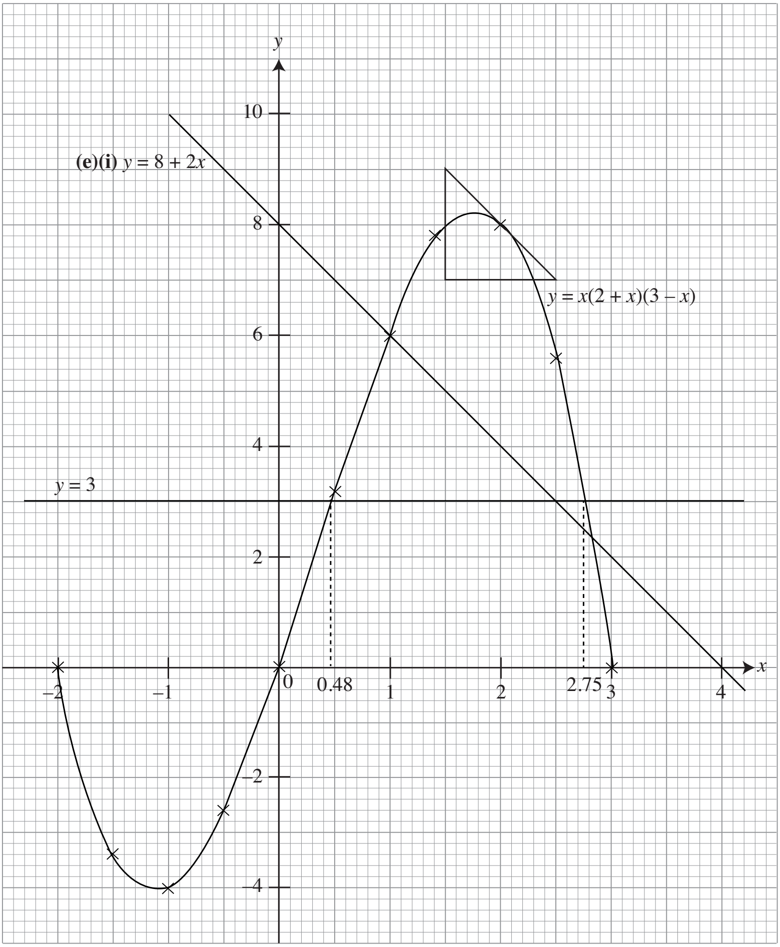
(e) **(i)** On the same axes, draw the graph of $y = 8 - 2x$ for values of x in the range $-1 \leqslant x \leqslant 4$.

(ii) Write down and simplify the cubic equation which is satisfied by the values of x at the points where the two graphs intersect.

Solution

(a) $p = 1(2 + 1)(3 - 1)$
$= 6$
$q = 3(2 + 3)(3 - 3)$
$= 0$

(b)



(c) From the graph, $x = 0.48$ and 2.75 when $y = 3$.

(d) Gradient of tangent $= \dfrac{7-9}{2.5-1.5}$

$= -2$

(e) **(ii)** $x(2 + x)(3 - x) = 8 - 2x$

$x(6 + x - x^2) = 8 - 2x$

$6x + x^2 - x^3 = 8 - 2x$

$x^3 - x^2 - 8x + 8 = 0$

UNIT 1.8

Solutions of Equations and Inequalities

Quadratic Formula

1. To solve the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1

Solve the equation $3x^2 - 3x - 2 = 0$.

Solution

Determine the values of a, b and c.

$a = 3, b = -3, c = -2$

$$\begin{aligned} x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-2)}}{2(3)} \\ &= \frac{3 \pm \sqrt{9 - (-24)}}{6} \\ &= \frac{3 \pm \sqrt{33}}{6} \\ &= 1.46 \text{ or } -0.457 \text{ (to 3 s. f.)} \end{aligned}$$

Example 2

Using the quadratic formula, solve the equation $5x^2 + 9x - 4 = 0$.

Solution

In $5x^2 + 9x - 4 = 0$, $a = 5$, $b = 9$, $c = -4$.

$$x = \frac{-9 \pm \sqrt{9^2 - 4(5)(-4)}}{2(5)}$$

$$= \frac{-9 \pm \sqrt{161}}{10}$$

$$= 0.369 \text{ or } -2.17 \text{ (to 3 s.f.)}$$

Example 3

Solve the equation $6x^2 + 3x - 8 = 0$.

Solution

In $6x^2 + 3x - 8 = 0$, $a = 6$, $b = 3$, $c = -8$.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(6)(-8)}}{2(6)}$$

$$= \frac{-3 \pm \sqrt{201}}{12}$$

$$= 0.931 \text{ or } -1.43 \text{ (to 3 s.f.)}$$

Example 4

Solve the equation $\frac{1}{x+8} + \frac{3}{x-6} = 10$.

Solution

$$\frac{1}{x+8} + \frac{3}{x-6} = 10$$

$$\frac{(x-6)+3(x+8)}{(x+8)(x-6)} = 10$$

$$\frac{x-6+3x+24}{(x+8)(x-6)} = 10$$

$$\frac{4x+18}{(x+8)(x-6)} = 10$$

$$4x + 18 = 10(x + 8)(x - 6)$$

$$4x + 18 = 10(x^2 + 2x - 48)$$

$$2x + 9 = 10x^2 + 20x - 480$$

$$2x + 9 = 5(x^2 + 2x - 48)$$

$$2x + 9 = 5x^2 + 10x - 240$$

$$5x^2 + 8x - 249 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(5)(-249)}}{2(5)}$$

$$= \frac{-8 \pm \sqrt{5044}}{10}$$

$$= 6.30 \text{ or } -7.90 \text{ (to 3 s.f.)}$$

Example 5

A cuboid has a total surface area of 250 cm^2. Its width is 1 cm shorter than its length, and its height is 3 cm longer than the length. What is the length of the cuboid?

Solution

Let x represent the length of the cuboid.
Let $x - 1$ represent the width of the cuboid.
Let $x + 3$ represent the height of the cuboid.

$$\begin{aligned}\text{Total surface area} &= 2(x)(x+3) + 2(x)(x-1) + 2(x+3)(x-1)\\ 250 &= 2(x^2+3x) + 2(x^2-x) + 2(x^2+2x-3) \quad \text{(Divide the equation by 2)}\\ 125 &= x^2+3x+x^2-x+x^2+2x-3\end{aligned}$$

$$3x^2 + 4x - 128 = 0$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-128)}}{2(3)}$$

$$= 5.90 \text{ (to 3 s.f.) or } -7.23 \text{ (rejected)}$$ (Reject –7.23 since the length cannot be less than 0)

∴ The length of the cuboid is 5.90 cm.

Completing the Square

2. To solve the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$:

Step 1: Change the coefficient of x^2 to 1,

i.e. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

Step 2: Bring $\frac{c}{a}$ to the right side of the equation,

i.e. $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step 3: Divide the coefficient of x by 2 and add the square of the result to both sides of the equation,

i.e. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

Step 4: Factorise and simplify.

i.e. $$\left(x+\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$= \pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Example 6

Solve the equation $x^2 - 4x - 8 = 0$.

Solution

$$x^2 - 4x - 8 = 0$$

$$x^2 - 2(2)(x) + 2^2 = 8 + 2^2$$

$$(x-2)^2 = 12$$

$$x - 2 = \pm\sqrt{12}$$

$$x = \pm\sqrt{12} + 2$$

$$= 5.46 \text{ or } -1.46 \text{ (to 3 s.f.)}$$

Example 7

Using the method of completing the square, solve the equation $2x^2 + x - 6 = 0$.

Solution

$$
\begin{aligned}
2x^2 + x - 6 &= 0 \\
x^2 + \frac{1}{2}x - 3 &= 0 \\
x^2 + \frac{1}{2}x &= 3 \\
x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 &= 3 + \left(\frac{1}{4}\right)^2 \\
\left(x + \frac{1}{4}\right)^2 &= \frac{49}{16} \\
x + \frac{1}{4} &= \pm\frac{7}{4} \\
x &= -\frac{1}{4} \pm \frac{7}{4} \\
&= 1.5 \text{ or } -2
\end{aligned}
$$

Example 8

Solve the equation $2x^2 - 8x - 24$ by completing the square.

Solution

$$
\begin{aligned}
2x^2 - 8x - 24 &= 0 \\
x^2 - 4x - 12 &= 0 \\
x^2 - 2(2)x + 2^2 &= 12 + 2^2 \\
(x - 2)^2 &= 16 \\
x - 2 &= \pm 4 \\
x &= 6 \text{ or } -2
\end{aligned}
$$

Solving Simultaneous Equations

3. **Elimination method** is used by making the coefficient of one of the variables in the two equations the same. Either add or subtract to form a single linear equation of only one unknown variable.

Example 9

Solve the simultaneous equations

$$2x + 3y = 15,$$
$$-3y + 4x = 3.$$

Solution

$2x + 3y = 15$ —— (1)

$-3y + 4x = 3$ —— (2)

(1) + (2):

$$(2x + 3y) + (-3y + 4x) = 18$$
$$6x = 18$$
$$x = 3$$

Substitute $x = 3$ into (1):

$$2(3) + 3y = 15$$
$$3y = 15 - 6$$
$$y = 3$$

$\therefore x = 3, y = 3$

Example 10

Using the method of elimination, solve the simultaneous equations

$$5x + 2y = 10,$$
$$4x + 3y = 1.$$

Solution

$5x + 2y = 10$ —— (1)

$4x + 3y = 1$ —— (2)

(1) × 3: $15x + 6y = 30$ —— (3)

(2) × 2: $8x + 6y = 2$ —— (4)

(3) – (4): $7x = 28$

$x = 4$

Substitute $x = 4$ into (2):

$4(4) + 3y = 1$

$16 + 3y = 1$

$3y = -15$

$y = -5$

$\therefore x = 4, y = -5$

4. **Substitution method** is used when we make one variable the subject of an equation and then we substitute that into the other equation to solve for the other variable.

Example 11

Solve the simultaneous equations

$$2x - 3y = -2,$$
$$y + 4x = 24.$$

Solution

$2x - 3y = -2$ —— (1)

$y + 4x = 24$ —— (2)

From (1),

$$x = \frac{-2+3y}{2}$$
$$= -1 + \frac{3}{2}y \text{ —— (3)}$$

Substitute (3) into (2):

$$y + 4\left(-1+\frac{3}{2}y\right) = 24$$
$$y - 4 + 6y = 24$$
$$7y = 28$$
$$y = 4$$

Substitute $y = 4$ into (3):

$$x = -1 + \frac{3}{2}y$$
$$= -1 + \frac{3}{2}(4)$$
$$= -1 + 6$$
$$= 5$$

$\therefore x = 5, y = 4$

Example 12

Using the method of substitution, solve the simultaneous equations

$$5x + 2y = 10,$$
$$4x + 3y = 1.$$

Solution

$5x + 2y = 10$ —— (1)

$4x + 3y = 1$ —— (2)

From (1),

$2y = 10 - 5x$

$y = \dfrac{10 - 5x}{2}$ —— (3)

Substitute (3) into (2):

$$4x + 3\left(\frac{10 - 5x}{2}\right) = 1$$

$$8x + 3(10 - 5x) = 2$$
$$8x + 30 - 15x = 2$$
$$7x = 28$$
$$x = 4$$

Substitute $x = 4$ into (3):

$$y = \frac{10 - 5(4)}{2}$$
$$= -5$$

$\therefore\ x = 4,\ y = -5$

Inequalities

5. To solve an inequality, we multiply or divide both sides by a positive number without having to reverse the inequality sign,

i.e. if $x \geqslant y$ and $c > 0$, then $cx \geqslant cy$ and $\frac{x}{c} \geqslant \frac{y}{c}$

and if $x > y$ and $c > 0$, then $cx > cy$ and $\frac{x}{c} > \frac{y}{c}$.

6. To solve an inequality, we reverse the inequality sign if we multiply or divide both sides by a negative number,

i.e. if $x \geqslant y$ and $d < 0$, then $dx \leqslant dy$ and $\frac{x}{d} \leqslant \frac{y}{d}$

and if $x > y$ and $d < 0$, then $dx < dy$ and $\frac{x}{d} < \frac{y}{d}$.

Solving Inequalities

7. To solve linear inequalities such as $px + q < r$ whereby $p \neq 0$,

$x < \frac{r-q}{p}$, if $p > 0$

$x > \frac{r-q}{p}$, if $p < 0$

Example 13

Solve the inequality $2 - 5x \geqslant 3x - 14$.

Solution

$$\begin{aligned} 2 - 5x &\geqslant 3x - 14 \\ -5x - 3x &\geqslant -14 - 2 \\ -8x &\geqslant -16 \\ x &\leqslant 2 \end{aligned}$$

Example 14

Given that $\frac{x+3}{5}+1<\frac{x+2}{2}-\frac{x+1}{4}$, find

(a) the least integer value of x,

(b) the smallest value of x such that x is a prime number.

Solution

$$\frac{x+3}{5}+1<\frac{x+2}{2}-\frac{x+1}{4}$$

$$\frac{x+3+5}{5}<\frac{2(x+2)-(x+1)}{4}$$

$$\frac{x+8}{5}<\frac{2x+4-x-1}{4}$$

$$\frac{x+8}{5}<\frac{x+3}{4}$$

$$4(x+8)<5(x+3)$$

$$4x+32<5x+15$$

$$-x<-17$$

$$x>17$$

(a) The least integer value of x is 18.

(b) The smallest value of x such that x is a prime number is 19.

Example 15

Given that $1 \leqslant x \leqslant 4$ and $3 \leqslant y \leqslant 5$, find

(a) the largest possible value of $y^2 - x$,

(b) the smallest possible value of

(i) $\dfrac{y^2}{x}$,

(ii) $(y - x)^2$.

Solution

(a) Largest possible value of $y^2 - x = 5^2 - 1^2$

$$= 25 - 1$$
$$= 24$$

(b) **(i)** Smallest possible value of $\dfrac{y^2}{x} = \dfrac{3^2}{4}$

$$= 2\frac{1}{4}$$

(ii) Smallest possible value of $(y - x)^2 = (3 - 3)^2$

$$= 0$$

UNIT 1.9

Applications of Mathematics in Practical Situations

Hire Purchase

1. Expensive items may be paid through hire purchase, where the full cost is paid over a given period of time. The hire purchase price is usually greater than the actual cost of the item. Hire purchase price comprises an initial deposit and regular instalments. Interest is charged along with these instalments.

Example 1

A sofa set costs \$4800 and can be bought under a hire purchase plan. A 15% deposit is required and the remaining amount is to be paid in 24 monthly instalments at a simple interest rate of 3% per annum. Find

(i) the amount to be paid in instalment per month,

(ii) the percentage difference in the hire purchase price and the cash price.

Solution

(i) Deposit $= \frac{15}{100} \times \$4800$

$= \$720$

Remaining amount $= \$4800 - \$720$

$= \$4080$

Amount of interest to be paid in 2 years $= \frac{3}{100} \times \$4080 \times 2$ (24 months = 2 years)

$= \$244.80$

Total amount to be paid in monthly instalments $= \$4080 + \$244.80$

$= \$4324.80$

Amount to be paid in instalment per month $= \$4324.80 \div 24$

$= \$180.20$

\$180.20 has to be paid in instalment per month.

(ii) Hire purchase price = \$720 + \$4324.80

= \$5044.80

$$\text{Percentage difference} = \frac{\text{Hire purchase price} - \text{Cash price}}{\text{Cash price}} \times 100\%$$

$$= \frac{5044.80 - 4800}{4800} \times 100\%$$

$$= 5.1\%$$

∴ The percentage difference in the hire purchase price and cash price is 5.1%.

Simple Interest

2. To calculate simple interest, we use the formula

$$I = \frac{PRT}{100},$$

where I = simple interest,

P = principal amount,

R = rate of interest per annum,

T = period of time in years.

Example 2

A sum of \$500 was invested in an account which pays simple interest per annum. After 4 years, the amount of money increased to \$550. Calculate the interest rate per annum.

Solution

$$\frac{500(R)4}{100} = 50$$

$$R = 2.5$$

The interest rate per annum is 2.5%.

Compound Interest

3. Compound interest is the interest accumulated over a given period of time at a given rate when each successive interest payment is added to the principal amount.

4. To calculate compound interest, we use the formula

$$A = P\left(1 + \frac{R}{100}\right)^n,$$

where A = total amount after n units of time,
P = principal amount,
R = rate of interest per unit time,
n = number of units of time.

Example 3

Yvonne deposits \$5000 in a bank account which pays 4% per annum compound interest. Calculate the total interest earned in 5 years, correct to the nearest dollar.

Solution

$$\begin{aligned} \text{Interest earned} &= P\left(1 + \frac{R}{100}\right)^n - P \\ &= 5000\left(1 + \frac{4}{100}\right)^5 - 5000 \\ &= \$1083 \end{aligned}$$

Example 4

Brian wants to place \$10 000 into a fixed deposit account for 3 years. Bank X offers a simple interest rate of 1.2% per annum and Bank Y offers an interest rate of 1.1% compounded annually. Which bank should Brian choose to yield a better interest?

Solution

Interest offered by Bank X:

$$\begin{aligned} I &= \frac{PRT}{100} \\ &= \frac{(10\,000)(1.2)(3)}{100} \\ &= \$360 \end{aligned}$$

Interest offered by Bank Y:

$$\begin{aligned} I &= P\left(1+\frac{R}{100}\right)^n - P \\ &= 10\,000\left(1+\frac{1.1}{100}\right)^3 - 10\,000 \\ &= \$333.64 \text{ (to 2 d.p.)} \end{aligned}$$

$\therefore$ Brian should choose Bank X.

Money Exchange

5. To change local currency to foreign currency, a given unit of the local currency is multiplied by the exchange rate.

e.g. To change Singapore dollars to foreign currency,
Foreign currency = Singapore dollars × Exchange rate

Example 5

Mr Lim exchanged S\$800 for Australian dollars. Given S\$1 = A\$0.9611, how much did he receive in Australian dollars?

Solution

$800 \times 0.9611 = 768.88$

Mr Lim received A\$768.88.

Example 6

A tourist wanted to change S$500 into Japanese Yen. The exchange rate at that time was ¥100 = S$1.0918. How much will he receive in Japanese Yen?

Solution

$$\frac{500}{1.0918} \times 100 = 45\ 795.933$$

$$\approx 45\ 795.93$$ (Always leave answers involving money to the nearest cent unless stated otherwise)

He will receive ¥45 795.93.

6. To convert foreign currency to local currency, a given unit of the foreign currency is divided by the exchange rate.
e.g. To change foreign currency to Singapore dollars,
Singapore dollars = Foreign currency ÷ Exchange rate

Example 7

Sarah buys a dress in Thailand for 200 Baht. Given that S$1 = 25 Thai baht, how much does the dress cost in Singapore dollars?

Solution

$200 \div 25 = 8$

The dress costs S$8.

Profit and Loss

7. Profit = Selling price – Cost price

8. Loss = Cost price – Selling price

Example 8

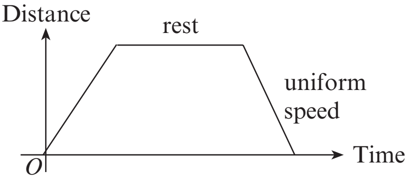
Mrs Lim bought a piece of land and sold it a few years later. She sold the land at \$3 million at a loss of 30%. How much did she pay for the land initially? Give your answer correct to 3 significant figures.

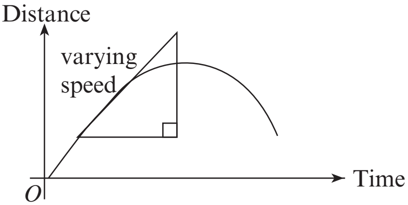
Solution

$$3\ 000\ 000 \times \frac{100}{70} = 4\ 290\ 000 \text{ (to 3 s.f.)}$$

Mrs Lim initially paid \$4 290 000 for the land.

Distance-Time Graphs





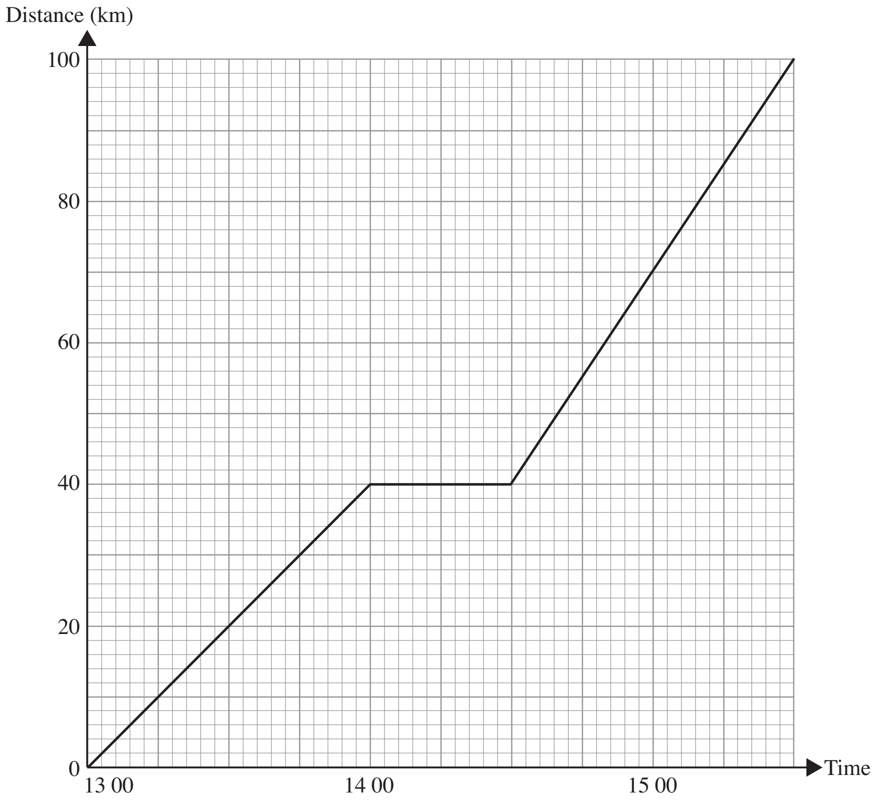
9. The gradient of a distance-time graph gives the speed of the object.

10. A straight line indicates motion with uniform speed.
A curve indicates motion with varying speed.
A straight line parallel to the time-axis indicates that the object is stationary.

11. Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

Example 9

The following diagram shows the distance-time graph for the journey of a motorcyclist.



(a) What was the distance covered in the first hour?

(b) Find the speed in km/h during the last part of the journey.

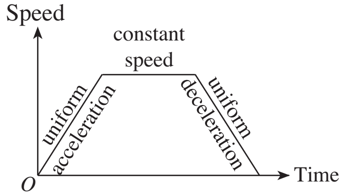
Solution

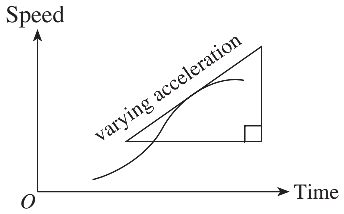
(a) Distance = 40 km

(b) $$\text{Speed} = \frac{100 - 40}{1}$$

$$= 60 \text{ km/h}$$

Speed-Time Graphs





12. The gradient of a speed-time graph gives the acceleration of the object.

13. A straight line indicates motion with uniform acceleration.
A curve indicates motion with varying acceleration.
A straight line parallel to the time-axis indicates that the object is moving with uniform speed.

14. Total distance covered in a given time = Area under the graph

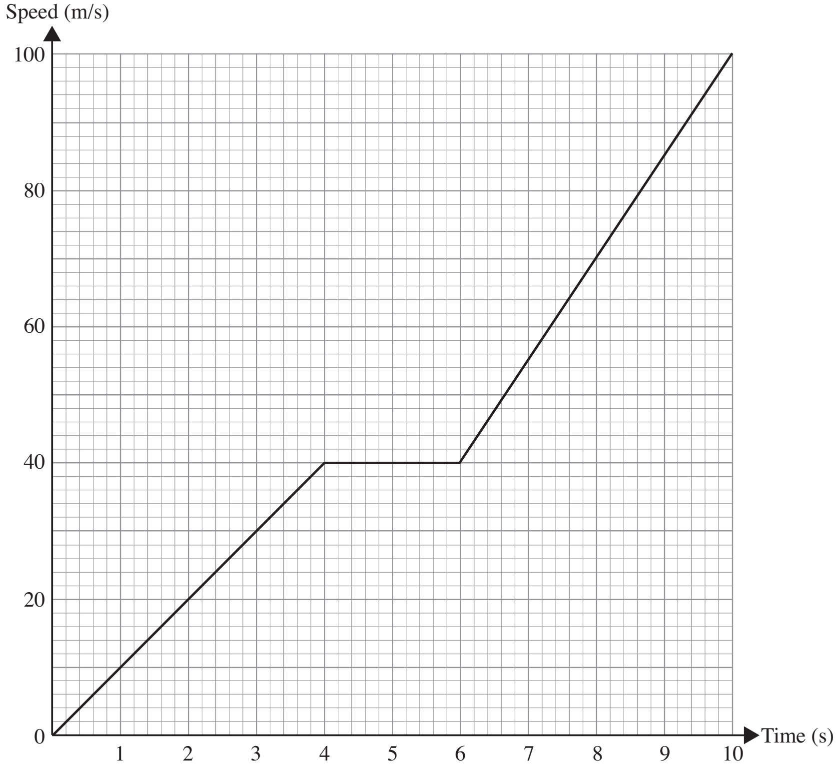
Example 10

The diagram below shows the speed-time graph of a bullet train journey for a period of 10 seconds.

(a) Find the acceleration during the first 4 seconds.

(b) How many seconds did the car maintain a constant speed?

(c) Calculate the distance travelled during the last 4 seconds.



Solution

(a) Acceleration $= \dfrac{40}{4}$

$= 10 \text{ m/s}^2$

(b) The car maintained at a constant speed for 2 s.

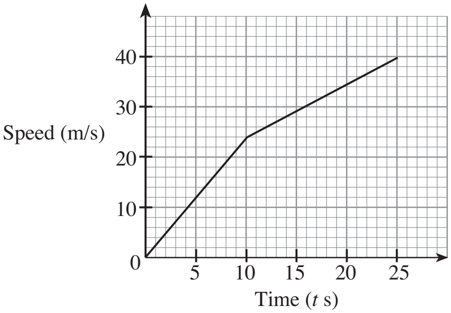
(c) Distance travelled = Area of trapezium

$= \dfrac{1}{2}(100 + 40)(4)$

$= 280 \text{ m}$

Example 11

The diagram shows the speed-time graph of the first 25 seconds of a journey.



Find

(i) the speed when $t = 15$,

(ii) the acceleration during the first 10 seconds,

(iii) the total distance travelled in the first 25 seconds.

Solution

(i) When $t = 15$, speed = 29 m/s.

(ii) Acceleration during the first 10 seconds $= \dfrac{24 - 0}{10 - 0}$

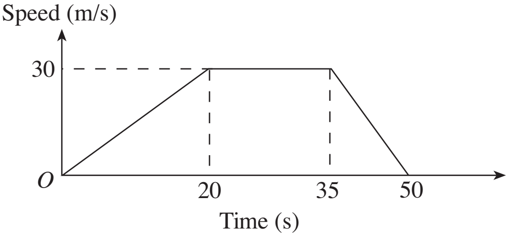
$= 2.4$ m/s^2

(iii) Total distance travelled $= \dfrac{1}{2}(10)(24) + \dfrac{1}{2}(24 + 40)(15)$

$= 600$ m

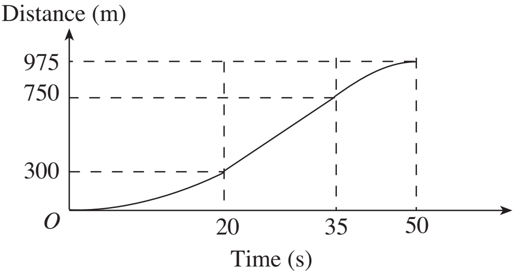
Example 12

Given the following speed-time graph, sketch the corresponding distance-time graph and acceleration-time graph.

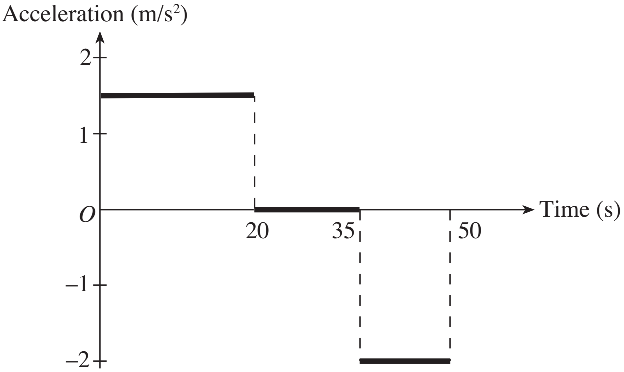


Solution

The distance-time graph is as follows:

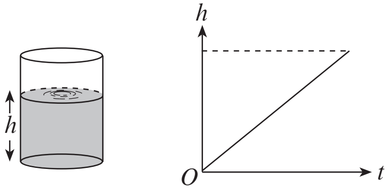


The acceleration-time graph is as follows:

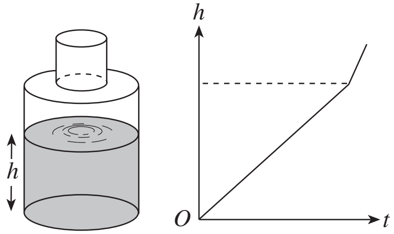


Water Level – Time Graphs

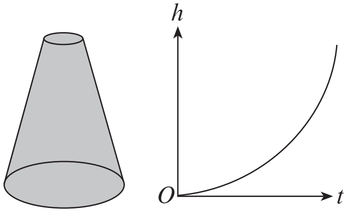
15. If the container is a cylinder as shown, the rate of the water level increasing with time is given as:



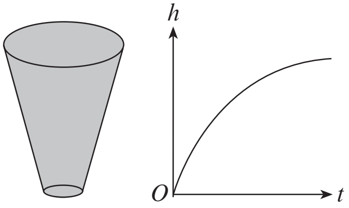
16. If the container is a bottle as shown, the rate of the water level increasing with time is given as:



17. If the container is an inverted funnel bottle as shown, the rate of the water level increasing with time is given as:



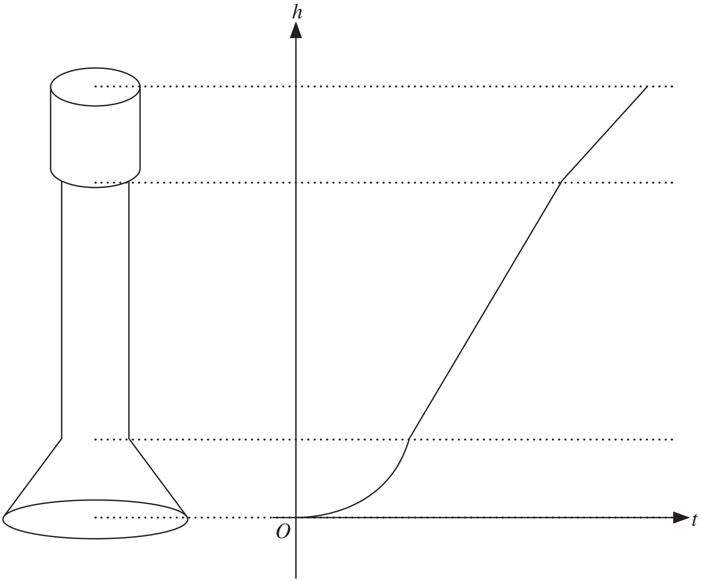
18. If the container is a funnel bottle as shown, the rate of the water level increasing with time is given as:



Example 13

Water is poured at a constant rate into the container below. Sketch the graph of water level (h) against time (t).

Solution



UNIT

1.10

Set Language and Notation

(not included for NA)

Definitions

1. A set is a collection of objects such as letters of the alphabet, people, etc. The objects in a set are called members or elements of that set.

2. A finite set is a set which contains a countable number of elements.

3. An infinite set is a set which contains an uncountable number of elements.

4. A universal set ξ is a set which contains all the available elements.

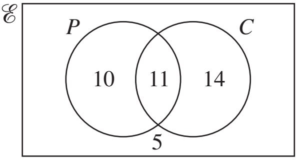
5. The empty set Ø or null set { } is a set which contains no elements.

Specifications of Sets

6. A set may be specified by listing all its members. This is only for finite sets. We list names of elements of a set, separate them by commas and enclose them in brackets, e.g. {2, 3, 5, 7}.

7. A set may be specified by stating a property of its elements, e.g. $\{x$: x is an even number greater than $3\}$.

8. A set may be specified by the use of a Venn diagram.
e.g.



For example, the Venn diagram above represents
ξ = {students in the class},
P = {students who study Physics},
C = {students who study Chemistry}.

From the Venn diagram,
10 students study Physics only,
14 students study Chemistry only,
11 students study both Physics and Chemistry,
5 students do not study either Physics or Chemistry.

Elements of a Set

9. a $\in Q$ means that a is an element of *Q*.
b $\notin Q$ means that b is not an element of *Q*.

10. n(*A*) denotes the number of elements in set *A*.

Example 1

ξ = {x: x is an integer such that $1 \leqslant x \leqslant 15$}
P = {x : x is a prime number},
Q = {x : x is divisible by 2},
R = {x : x is a multiple of 3}.

(a) List the elements in *P* and *R*.
(b) State the value of n(*Q*).

Solution

(a) $P = \{2, 3, 5, 7, 11, 13\}$
$R = \{3, 6, 9, 12, 15\}$
(b) $Q = \{2, 4, 6, 8, 10, 12, 14\}$
$n(Q) = 7$

Equal Sets

11. If two sets contain the exact same elements, we say that the two sets are equal sets. For example, if $A = \{1, 2, 3\}$, $B = \{3, 1, 2\}$ and $C = \{a, b, c\}$, then A and B are equal sets but A and C are not equal sets.

Subsets

12. $A \subseteq B$ means that A is a subset of B.
Every element of set A is also an element of set B.

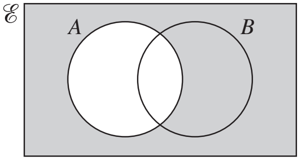
13. $A \subset B$ means that A is a proper subset of B.
Every element of set A is also an element of set B, but A cannot be equal to B.

14. $A \nsubseteq B$ means A is not a subset of B.

15. $A \not\subset B$ means A is not a proper subset of B.

Complement Sets

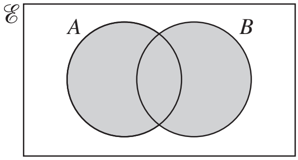
16. A' denotes the complement of a set A relative to a universal set ξ.
It is the set of all elements in ξ except those in A.



The shaded region in the diagram shows A'.

Union of Two Sets

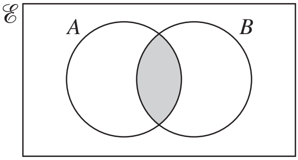
17. The union of two sets A and B, denoted as $A \cup B$, is the set of elements which belong to set A or set B or both.



The shaded region in the diagram shows $A \cup B$.

Intersection of Two Sets

18. The intersection of two sets A and B, denoted as $A \cap B$, is the set of elements which belong to both set A and set B.



The shaded region in the diagram shows $A \cap B$.

Example 2

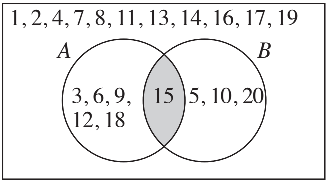
$\xi = \{x: x \text{ is an integer such that } 1 \leqslant x \leqslant 20\}$
$A = \{x : x \text{ is a multiple of } 3\}$
$B = \{x : x \text{ is divisible by } 5\}$

(a) Draw a Venn diagram to illustrate this information.
(b) List the elements contained in the set $A \cap B$.

Solution

$A = \{3, 6, 9, 12, 15, 18\}$
$B = \{5, 10, 15, 20\}$

(a)

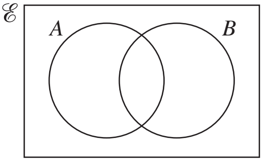


(b) $A \cap B = \{15\}$

Example 3

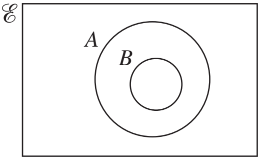
Shade the set indicated in each of the following Venn diagrams.

(a)



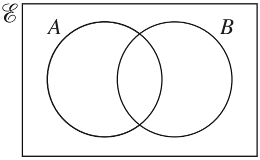
$A \cap B'$

(b)



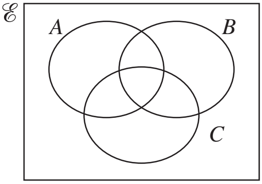
$A' \cap B$

(c)



$(A \cap B)'$

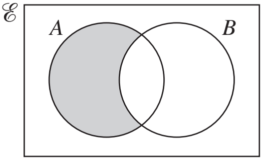
(d)



$A \cap C \cap B$

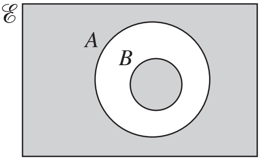
Solution

(a)



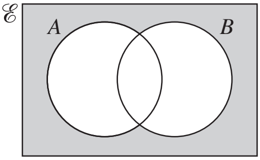
$A \cap B'$

(b)



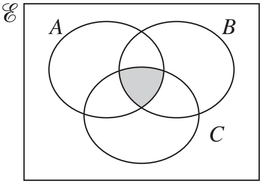
$A' \cap B$

(c)



$(A \cap B)'$

(d)

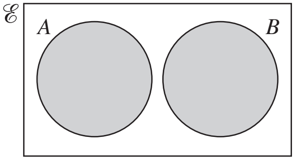


$A \cap C \cap B$

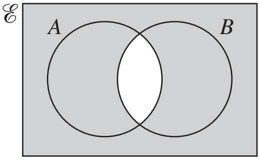
Example 4

Write the set notation for the sets shaded in each of the following Venn diagrams.

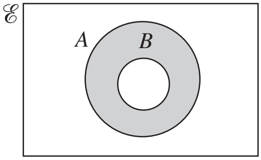
(a)



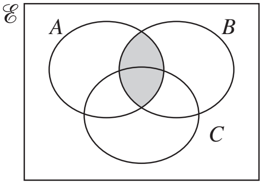
(b)



(c)



(d)



Solution

(a) $A \cup B$
(b) $(A \cap B)'$
(c) $A \cap B'$
(d) $A \cap B$

Example 5

$\xi = \{x \text{ is an integer} : -2 \leqslant x \leqslant 5\}$
$P = \{x : -2 < x < 3\}$
$Q = \{x : 0 < x \leqslant 4\}$

List the elements in
(a) P',
(b) $P \cap Q$,
(c) $P \cup Q$.

Solution

$\xi = \{-2, -1, 0, 1, 2, 3, 4, 5\}$
$P = \{-1, 0, 1, 2\}$
$Q = \{1, 2, 3, 4\}$

(a) $P' = \{-2, 3, 4, 5\}$
(b) $P \cap Q = \{1, 2\}$
(c) $P \cup Q = \{-1, 0, 1, 2, 3, 4\}$

Example 6

$\xi = \{x : x$ is a real number: $x < 30\}$
$A = \{x : x$ is a prime number$\}$
$B = \{x : x$ is a multiple of $3\}$
$C = \{x : x$ is a multiple of $4\}$

(a) Find $A \cap B$.
(b) Find $A \cap C$.
(c) Find $B \cap C$.

Solution

(a) $A \cap B = \{3\}$
(b) $A \cap C = \emptyset$
(c) $B \cap C = \{12, 24\}$ (Common multiples of 3 and 4 that are below 30)

Example 7

It is given that
ξ = {people on a bus}
A = {male commuters}
B = {students}
C = {commuters below 21 years old}

(a) Express in set notation, students who are below 21 years old.
(b) Express in words, $A' \cap B = \emptyset$.

Solution

(a) $B \subset C$ or $B \cap C$
(b) There are no female commuters who are students.

UNIT 1.11

Matrices

(not included for NA)

Matrix

1. A matrix is a rectangular array of numbers.

2. An example is $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -5 & 8 & 9 \\ 7 & 6 & -1 & 0 \end{pmatrix}$.

This matrix has 3 rows and 4 columns. We say that it has an order of 3 by 4.

3. In general, a matrix is defined by an order of $r \times c$, where r is the number of rows and c is the number of columns.
1, 2, 3, 4, 0, –5, … , 0 are called the elements of the matrix.

Row Matrix

4. A row matrix is a matrix that has exactly one row.

5. Examples of row matrices are $\begin{pmatrix} 12 & 4 & 3 \end{pmatrix}$ and $\begin{pmatrix} 7 & 5 \end{pmatrix}$.

6. The order of a row matrix is $1 \times c$, where c is the number of columns.

Column Matrix

7. A column matrix is a matrix that has exactly one column.

8. Examples of column matrices are $\begin{pmatrix} 0.5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$.

9. The order of a column matrix is $r \times 1$, where r is the number of rows.

Square Matrix

10. A square matrix is a matrix that has exactly the same number of rows and columns.

11. Examples of square matrices are $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $\begin{pmatrix} 6 & 0 & -2 \\ 5 & 8 & 4 \\ 0 & 3 & 9 \end{pmatrix}$.

12. Matrices such as $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -4 \end{pmatrix}$ are known as diagonal matrices as all the elements except those in the leading diagonal are zero.

Zero Matrix or Null Matrix

13. A zero matrix or null matrix is one where every element is equal to zero.

14. Examples of zero matrices or null matrices are $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

15. A zero matrix or null matrix is usually denoted by **0**.

Identity Matrix

16. An identity matrix is usually represented by the symbol **I**. All elements in its leading diagonal are ones while the rest are zeros.

e.g. 2×2 identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

3×3 identity matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

17. Any matrix **P** when multiplied by an identity matrix **I** will result in itself, i.e. **PI = IP = P**

Addition and Subtraction of Matrices

18. Matrices can only be added or subtracted if they are of the same order.

19. If there are two matrices **A** and **B**, both of order $r \times c$, then the addition of **A** and **B** is the addition of each element of **A** with its corresponding element of **B**,

i.e. $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$

and $\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$.

Example 1

Given that $\mathbf{P} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ and $\mathbf{P} + \mathbf{Q} = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 5 & 5 \end{pmatrix}$, find **Q**.

Solution

$$\mathbf{Q} = \begin{pmatrix} 3 & 2 & 5 \\ 4 & 5 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

Multiplication of a Matrix by a Real Number

20. The product of a matrix by a real number k is a matrix with each of its elements multiplied by k,

i.e. $k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

Multiplication of Two Matrices

21. Matrix multiplication is only possible when the number of columns in the matrix on the left is equal to the number of rows in the matrix on the right.

22. In general, multiplying a $m \times n$ matrix by a $n \times p$ matrix will result in a $m \times p$ matrix.

23. Multiplication of matrices is not commutative, i.e. **AB** ≠ **BA**.

24. Multiplication of matrices is associative, i.e. **A**(**BC**) = (**AB**)**C**, provided that the multiplication can be carried out.

Example 2

Given that $\mathbf{A} = \begin{pmatrix} 5 & 6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, find **AB**.

Solution

$$\begin{aligned}\mathbf{AB} &= \begin{pmatrix} 5 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 5\times1+6\times3 & 5\times2+6\times4 \end{pmatrix} \\ &= \begin{pmatrix} 23 & 34 \end{pmatrix}\end{aligned}$$

Example 3

Given $\mathbf{P} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 & 5 \\ 3 & 4 \\ 1 & 2 \end{pmatrix}$, find **PQ**.

Solution

$$\begin{aligned}\mathbf{PQ} &= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 5 \\ 3 & 4 \\ 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1\times2+2\times3+3\times1 & 1\times5+2\times4+3\times2 \\ 2\times2+3\times3+4\times1 & 2\times5+3\times4+4\times2 \end{pmatrix} \\ &= \begin{pmatrix} 11 & 19 \\ 17 & 30 \end{pmatrix}\end{aligned}$$

Example 4

At a school's food fair, there are 3 stalls each selling 3 different flavours of pancake – chocolate, cheese and red bean. The table illustrates the number of pancakes sold during the food fair.

	Stall 1	Stall 2	Stall 3
Chocolate	92	102	83
Cheese	86	73	56
Red bean	85	53	66

The price of each pancake is as follows:

Chocolate: \$1.10
Cheese: \$1.30
Red bean: \$0.70

(a) Write down two matrices such that, under matrix multiplication, the product indicates the total revenue earned by each stall. Evaluate this product.

(b) **(i)** Find $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 92 & 102 & 83 \\ 86 & 73 & 56 \\ 85 & 53 & 66 \end{pmatrix}$.

(ii) Explain what your answer to **(b)(i)** represents.

Solution

(a) $\begin{pmatrix} 1.10 & 1.30 & 0.70 \end{pmatrix}\begin{pmatrix} 92 & 102 & 83 \\ 86 & 73 & 56 \\ 85 & 53 & 66 \end{pmatrix} = \begin{pmatrix} 272.50 & 244.20 & 210.30 \end{pmatrix}$

(b) **(i)** $\begin{pmatrix} 263 & 228 & 205 \end{pmatrix}$

(ii) Each of the elements represents the total number of pancakes sold by each stall during the food fair.

Example 5

A BBQ caterer distributes 3 types of satay – chicken, mutton and beef to 4 families. The price of each stick of chicken, mutton and beef satay is \$0.32, \$0.38 and \$0.28 respectively.

Mr Wong orders 25 sticks of chicken satay, 60 sticks of mutton satay and 15 sticks of beef satay.

Mr Lim orders 30 sticks of chicken satay and 45 sticks of beef satay.

Mrs Tan orders 70 sticks of mutton satay and 25 sticks of beef satay.

Mrs Koh orders 60 sticks of chicken satay, 50 sticks of mutton satay and 40 sticks of beef satay.

(i) Express the above information in the form of a matrix **A** of order 4 by 3 and a matrix **B** of order 3 by 1 so that the matrix product **AB** gives the total amount paid by each family.

(ii) Evaluate **AB**.

(iii) Find the total amount earned by the caterer.

Solution

(i) $$\mathbf{A} = \begin{pmatrix} 25 & 60 & 15 \\ 30 & 0 & 45 \\ 0 & 70 & 25 \\ 60 & 50 & 40 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0.32 \\ 0.38 \\ 0.28 \end{pmatrix}$$

(ii) $$\mathbf{AB} = \begin{pmatrix} 25 & 60 & 15 \\ 30 & 0 & 45 \\ 0 & 70 & 25 \\ 60 & 50 & 40 \end{pmatrix} \begin{pmatrix} 0.32 \\ 0.38 \\ 0.28 \end{pmatrix} = \begin{pmatrix} 35 \\ 22.2 \\ 33.6 \\ 49.4 \end{pmatrix}$$

(iii) Total amount earned = \$35 + \$22.20 + \$33.60 + \$49.40

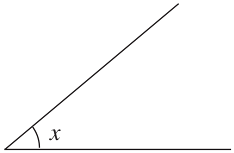
= \$140.20

UNIT 2.1 Angles, Triangles and Polygons

Types of Angles

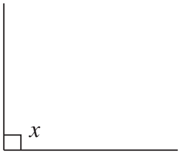
1. In a polygon, there may be four types of angles – acute angle, right angle, obtuse angle and reflex angle.

Acute angle



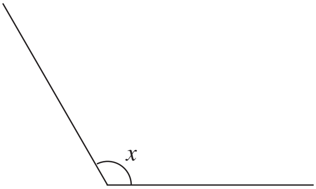
$x < 90°$

Right angle



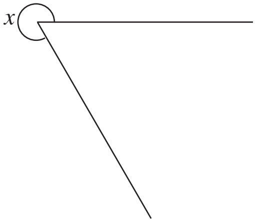
$x = 90°$

Obtuse angle



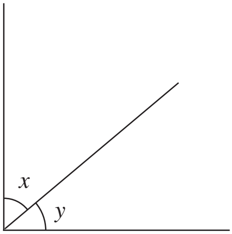
$90° < x < 180°$

Reflex angle



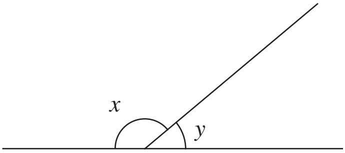
$180° < x < 360°$

2. If the sum of two angles is 90°, they are called complementary angles.



$\angle x + \angle y = 90°$

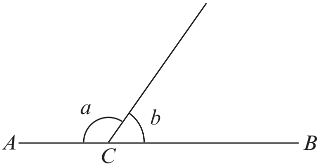
3. If the sum of two angles is 180°, they are called supplementary angles.



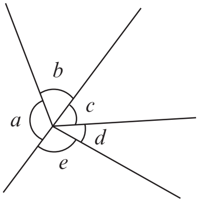
$\angle x + \angle y = 180°$

Geometrical Properties of Angles

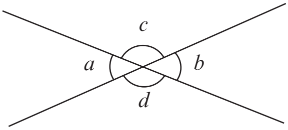
4. If *ACB* is a straight line, then $\angle a + \angle b = 180°$ (adj. ∠s on a str. line).



5. The sum of angles at a point is 360°, i.e. $\angle a + \angle b + \angle c + \angle d + \angle e = 360°$ (∠s at a pt.).



6. If two straight lines intersect, then $\angle a = \angle b$ and $\angle c = \angle d$ (vert. opp. ∠s).

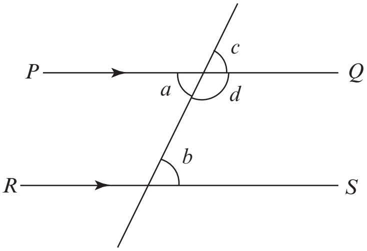


7. If the lines PQ and RS are parallel, then

$\angle a = \angle b$ (alt. $\angle$s),

$\angle c = \angle b$ (corr. $\angle$s),

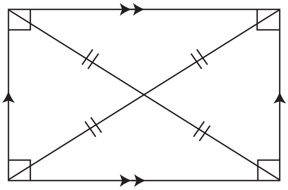
$\angle b + \angle d = 180°$ (int. $\angle$s).



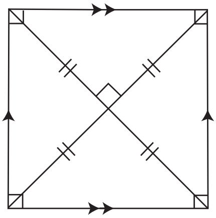
Properties of Special Quadrilaterals

8. The sum of the interior angles of a quadrilateral is 360°.

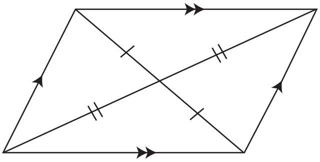
9. The diagonals of a rectangle bisect each other and are equal in length.



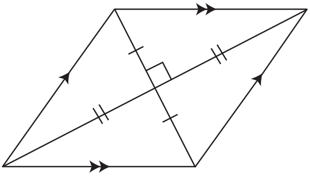
10. The diagonals of a square bisect each other at 90° and are equal in length. They bisect the interior angles.



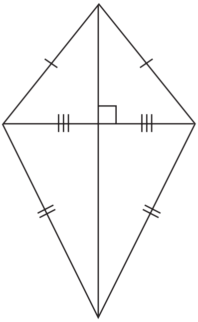
11. The diagonals of a parallelogram bisect each other.



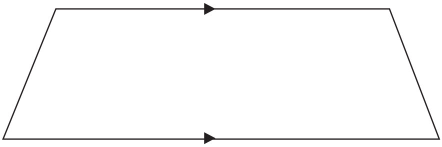
12. The diagonals of a rhombus bisect each other at 90°. They bisect the interior angles.



13. The diagonals of a kite cut each other at 90°. One of the diagonals bisects the interior angles.

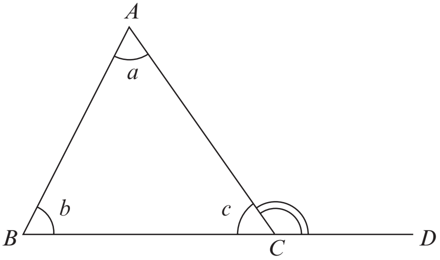


14. At least one pair of opposite sides of a trapezium are parallel to each other.



Geometrical Properties of Polygons

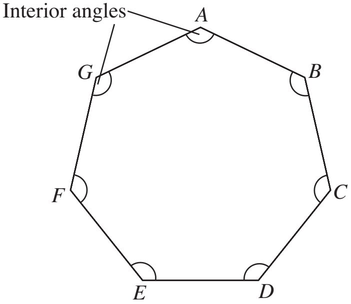
15. The sum of the interior angles of a triangle is 180°, i.e. $\angle a + \angle b + \angle c = 180°$ ($\angle$ sum of Δ).



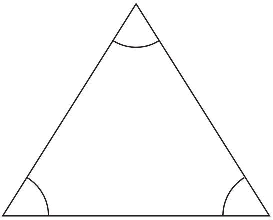
16. If the side BC of ΔABC is produced to D, then $\angle ACD = \angle a + \angle b$ (ext. $\angle$ of Δ).

17. The sum of the interior angles of an n-sided polygon is $(n - 2) \times 180°$.

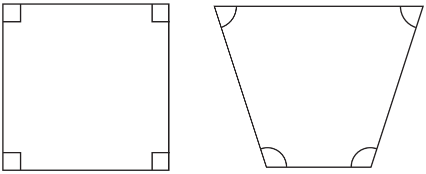
Each interior angle of a regular n-sided polygon = $\dfrac{(n-2)\times 180°}{n}$



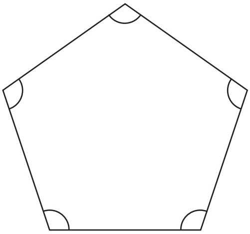
(a) A triangle is a polygon with 3 sides.
Sum of interior angles = $(3 - 2) \times 180° = 180°$



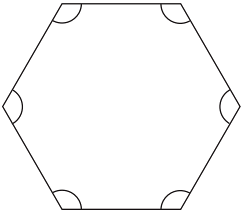
(b) A quadrilateral is a polygon with 4 sides.
Sum of interior angles = (4 – 2) × 180° = 360°



(c) A pentagon is a polygon with 5 sides.
Sum of interior angles = (5 – 2) × 180° = 540°

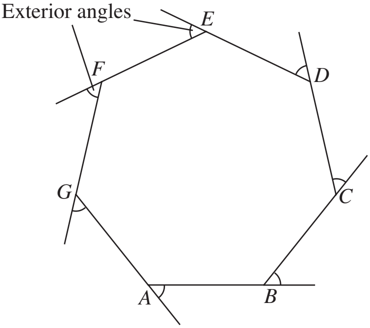


(d) A hexagon is a polygon with 6 sides.
Sum of interior angles = (6 – 2) × 180° = 720°



18. The sum of the exterior angles of an *n*-sided polygon is 360°.

Each exterior angle of a regular *n*-sided polygon = $\frac{360°}{n}$



Example 1

Three interior angles of a polygon are 145°, 120° and 155°. The remaining interior angles are 100° each. Find the number of sides of the polygon.

Solution

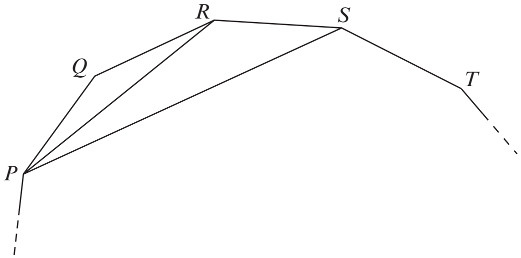
$$360° - (180° - 145°) - (180° - 120°) - (180° - 155°) = 360° - 35° - 60° - 25°$$
$$= 240°$$

$$\frac{240°}{(180° - 100°)} = 3$$

Total number of sides = 3 + 3
= 6

Example 2

The diagram shows part of a regular polygon with n sides. Each exterior angle of this polygon is 24°.



Find

(i) the value of n,

(ii) $P\hat{Q}R$,

(iii) $P\hat{R}S$,

(iv) $P\hat{S}R$.

Solution

(i) $$\text{Exterior angle} = \frac{360^\circ}{n}$$

$$24^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{24^\circ}$$

$$= 15$$

(ii) $$P\hat{Q}R = 180^\circ - 24^\circ$$

$$= 156^\circ$$

(iii) $$P\hat{R}Q = \frac{180^\circ - 156^\circ}{2}$$

$$= 12^\circ \text{ (base } \angle\text{s of isos. } \Delta)$$

$$P\hat{R}S = 156^\circ - 12^\circ$$

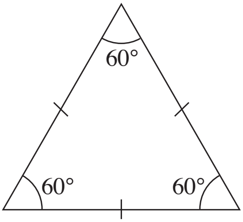
$$= 144^\circ$$

(iv) $$P\hat{S}R = 180^\circ - 156^\circ$$

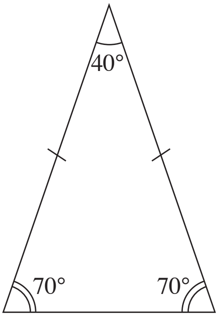
$$= 24^\circ \text{ (int. } \angle\text{s, } QR \,//\, PS)$$

Properties of Triangles

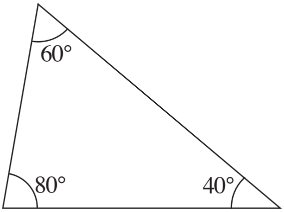
19. In an equilateral triangle, all three sides are equal. All three angles are the same, each measuring 60°.



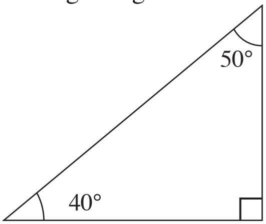
20. An isosceles triangle consists of two equal sides. Its two base angles are equal.



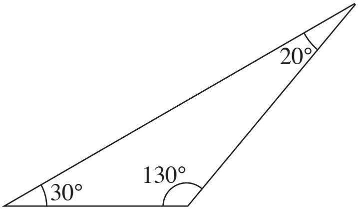
21. In an acute-angled triangle, all three angles are acute.



22. A right-angled triangle has one right angle.



23. An obtuse-angled triangle has one angle that is obtuse.

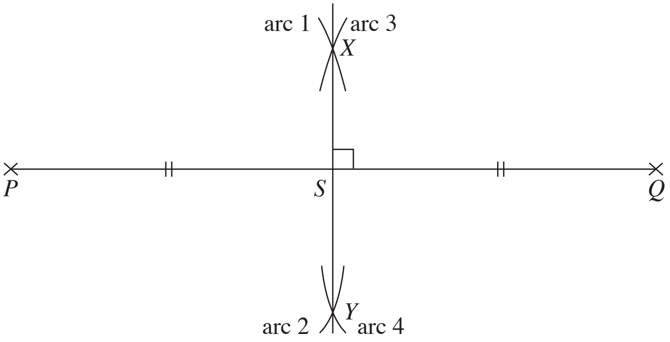


Perpendicular Bisector

24. If the line *XY* is the perpendicular bisector of a line segment *PQ*, then *XY* is perpendicular to *PQ* and *XY* passes through the midpoint of *PQ*.

Steps to construct a perpendicular bisector of line *PQ*:

1. Draw *PQ*.
2. Using a compass, choose a radius that is more than half the length of *PQ*.
3. Place the compass at *P* and mark arc 1 and arc 2 (one above and the other below the line *PQ*).
4. Place the compass at *Q* and mark arc 3 and arc 4 (one above and the other below the line *PQ*).
5. Join the two intersection points of the arcs to get the perpendicular bisector.



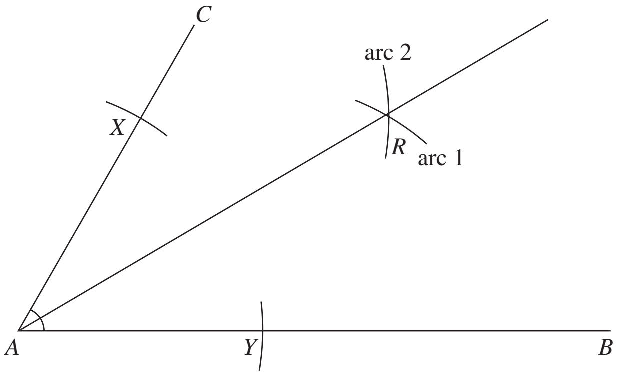
25. Any point on the perpendicular bisector of a line segment is equidistant from the two end points of the line segment.

Angle Bisector

26. If the ray AR is the angle bisector of $B\hat{A}C$, then $C\hat{A}R = R\hat{A}B$.

Steps to construct an angle bisector of $C\hat{A}B$:

1. Using a compass, choose a radius that is less than or equal to the length of AC.
2. Place the compass at A and mark two arcs (one on line AC and the other AB).
3. Mark the intersection points between the arcs and the two lines as X and Y.
4. Place compass at X and mark arc 2 (between the space of line AC and AB).
5. Place compass at Y and mark arc 1 (between the space of line AC and AB) that will intersect arc 2. Label the intersection point R.
6. Join R and A to bisect $C\hat{A}B$.



27. Any point on the angle bisector of an angle is equidistant from the two sides of the angle.

Example 3

Draw a quadrilateral $ABCD$ in which the base $AB = 3$ cm, $A\hat{B}C = 80°$, $BC = 4$ cm, $B\hat{A}D = 110°$ and $B\hat{C}D = 70°$.

(a) Measure and write down the length of CD.

(b) On your diagram, construct

  (i) the perpendicular bisector of AB,

  (ii) the bisector of angle ABC.

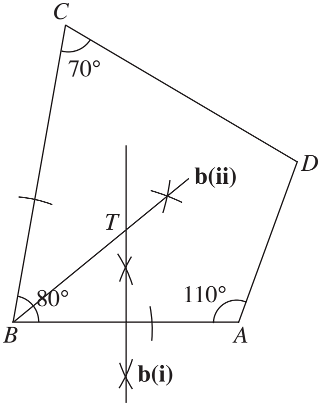
(c) These two bisectors meet at T.
Complete the statement below.

The point T is equidistant from the lines ____________ and ____________

and equidistant from the points ____________ and ____________ .

Solution

(a) $CD = 3.5$ cm

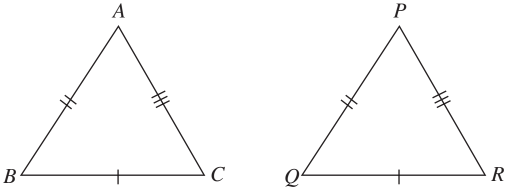


(c) The point T is equidistant from the lines AB and BC and equidistant from the points A and B.

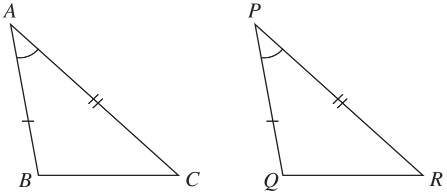
UNIT 2.2 Congruence and Similarity

Congruent Triangles

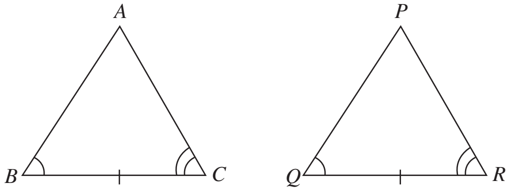
1. If $AB = PQ$, $BC = QR$ and $CA = RP$, then ΔABC is congruent to ΔPQR (SSS Congruence Test).



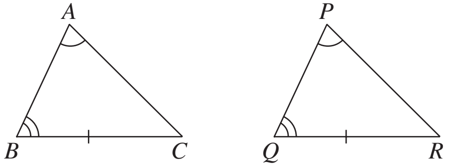
2. If $AB = PQ$, $AC = PR$ and $B\hat{A}C = Q\hat{P}R$, then ΔABC is congruent to ΔPQR (SAS Congruence Test).



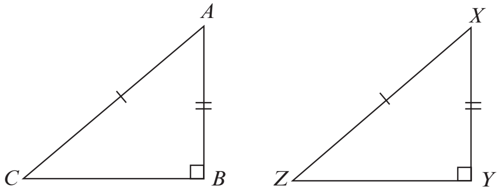
3. If $A\hat{B}C = P\hat{Q}R$, $A\hat{C}B = P\hat{R}Q$ and $BC = QR$, then ΔABC is congruent to ΔPQR (ASA Congruence Test).



If $B\hat{A}C = Q\hat{P}R$, $A\hat{B}C = P\hat{Q}R$ and $BC = QR$, then ΔABC is congruent to ΔPQR (AAS Congruence Test).



4. If $AC = XZ$, $AB = XY$ or $BC = YZ$, and $A\hat{B}C = X\hat{Y}Z = 90°$, then ΔABC is congruent to ΔXYZ (RHS Congruence Test).



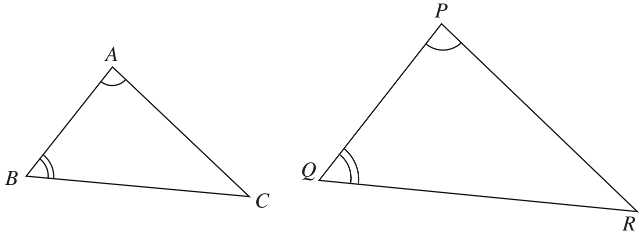
Similar Triangles

5. Two figures or objects are similar if:

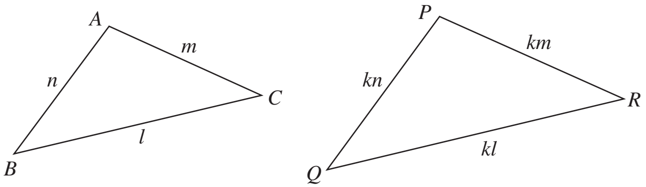
(a) the corresponding sides are proportional and,

(b) the corresponding angles are equal.

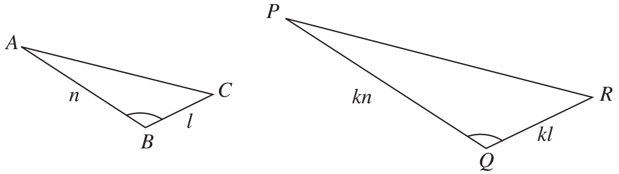
6. If $B\hat{A}C = Q\hat{P}R$ and $A\hat{B}C = P\hat{Q}R$, then ΔABC is similar to ΔPQR (AA Similarity Test).



7. If $\dfrac{PQ}{AB} = \dfrac{QR}{BC} = \dfrac{RP}{CA}$, then ΔABC is similar to ΔPQR (SSS Similarity Test).



8. If $\dfrac{PQ}{AB} = \dfrac{QR}{BC}$ and $A\hat{B}C = P\hat{Q}R$, then ΔABC is similar to ΔPQR (SAS Similarity Test).

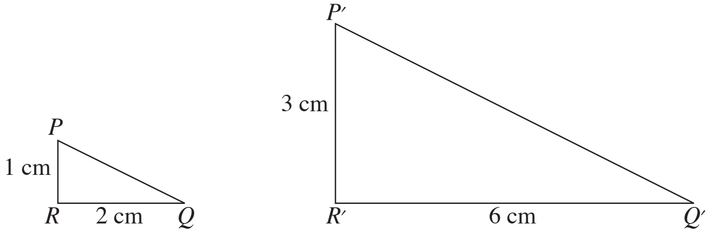


Scale Factor and Enlargement

9. Scale factor = $\dfrac{\text{Length of image}}{\text{Length of object}}$

10. We multiply the distance between each point in an object by a scale factor to produce an image. When the scale factor is greater than 1, the image produced is greater than the object. When the scale factor is between 0 and 1, the image produced is smaller than the object.

e.g. Taking ΔPQR as the object and $\Delta P'Q'R'$ as the image,
$\Delta P'Q'R'$ is an enlargement of ΔPQR with a scale factor of 3.



Scale factor = $\dfrac{P'R'}{PR} = \dfrac{R'Q'}{RQ} = 3$

If we take $\Delta P'Q'R'$ as the object and ΔPQR as the image,

ΔPQR is an enlargement of $\Delta P'Q'R'$ with a scale factor of $\dfrac{1}{3}$.

Scale factor = $\dfrac{PR}{P'R'} = \dfrac{RQ}{R'Q'} = \dfrac{1}{3}$

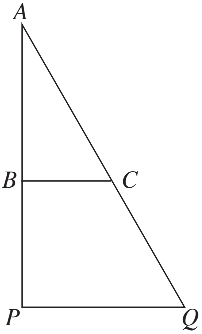
Similar Plane Figures

11. The ratio of the corresponding sides of two similar figures is $l_1 : l_2$.
The ratio of the area of the two figures is then $l_1^2 : l_2^2$.

e.g. ΔABC is similar to ΔAPQ.

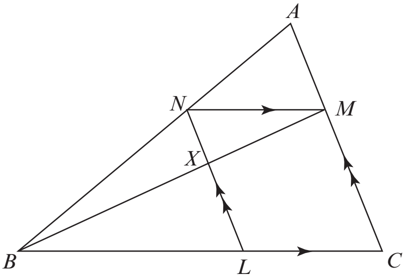
$$\frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ}$$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta APQ} = \left(\frac{AB}{AP}\right)^2 = \left(\frac{AC}{AQ}\right)^2 = \left(\frac{BC}{PQ}\right)^2$$



Example 1

In the figure, NM is parallel to BC and LN is parallel to CA.



(a) Prove that ΔANM is similar to ΔNBL.

(b) Given that $\frac{AN}{NB} = \frac{2}{3}$, find the numerical value of each of the following ratios.

(i) $\frac{\text{Area of } \Delta ANM}{\text{Area of } \Delta NBL}$

(ii) $\frac{NM}{BC}$

(iii) $\frac{\text{Area of trapezium } BNMC}{\text{Area of } \Delta ABC}$

(iv) $\frac{NX}{MC}$

Solution

(a) Since $A\hat{N}M = N\hat{B}L$ (corr. $\angle$s, $MN // LB$) and $N\hat{A}M = B\hat{N}L$ (corr. $\angle$s, $LN // MA$), ΔANM is similar to ΔNBL (AA Similarity Test).

(b) **(i)**

$$\frac{\text{Area of } \Delta ANM}{\text{Area of } \Delta NBL} = \left(\frac{AN}{NB}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

(ii) ΔANM is similar to ΔABC.

$$\therefore \frac{NM}{BC} = \frac{AN}{AB} = \frac{2}{5}$$

(iii) $$\frac{\text{Area of }\Delta ANM}{\text{Area of }\Delta ABC} = \left(\frac{NM}{BC}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\frac{\text{Area of trapezium } BNMC}{\text{Area of }\Delta ABC} = \frac{\text{Area of }\Delta ABC - \text{Area of }\Delta ANM}{\text{Area of }\Delta ABC} = \frac{25-4}{25} = \frac{21}{25}$$

(iv) ΔNMX is similar to ΔLBX and $MC = NL$.

$$\frac{NX}{LX} = \frac{NM}{LB} = \frac{NM}{BC-LC} = \frac{2}{3}$$

i.e. $$\frac{NX}{NL} = \frac{2}{5}$$

$$\therefore \frac{NX}{MC} = \frac{2}{5}$$

Example 2

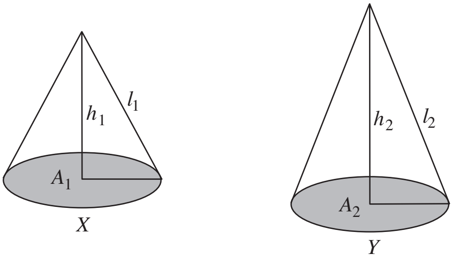
Triangle A and triangle B are similar. The length of one side of triangle A is $\frac{1}{4}$ the length of the corresponding side of triangle B. Find the ratio of the areas of the two triangles.

Solution

Let x be the length of triangle A.
Let $4x$ be the length of triangle B.

$$\begin{aligned}\frac{\text{Area of triangle } A}{\text{Area of triangle } B} &= \left(\frac{\text{Length of triangle } A}{\text{Length of triangle } B}\right)^2 \\ &= \left(\frac{x}{4x}\right)^2 \\ &= \frac{1}{16}\end{aligned}$$

Similar Solids



12. If X and Y are two similar solids, then the ratio of their lengths is equal to the ratio of their heights,

i.e. $\frac{l_1}{l_2} = \frac{h_1}{h_2}$.

13. If X and Y are two similar solids, then the ratio of their areas is given by

$$\frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{l_1}{l_2}\right)^2.$$

14. If X and Y are two similar solids, then the ratio of their volumes is given by

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{l_1}{l_2}\right)^3.$$

Example 3

The volumes of two glass spheres are 125 cm^3 and 216 cm^3. Find the ratio of the larger surface area to the smaller surface area.

Solution

Since $\frac{V_1}{V_2} = \frac{125}{216}$,

$$\frac{r_1}{r_2} = \sqrt[3]{\frac{125}{216}}$$

$$= \frac{5}{6}$$

$$\frac{A_1}{A_2} = \left(\frac{5}{6}\right)^2$$

$$= \frac{25}{36}$$

$\therefore$ The ratio is 36 : 25.

Example 4

The surface area of a small plastic cone is 90 cm^2. The surface area of a similar, larger plastic cone is 250 cm^2. Calculate the volume of the large cone if the volume of the small cone is 125 cm^3.

Solution

$$\frac{\text{Area of small cone}}{\text{Area of large cone}} = \frac{90}{250}$$

$$= \frac{9}{25}$$

$$\frac{\text{Area of small cone}}{\text{Area of large cone}} = \left(\frac{\text{Radius of small cone}}{\text{Radius of large cone}}\right)^2$$

$$= \frac{9}{25}$$

$$\frac{\text{Radius of small cone}}{\text{Radius of large cone}} = \frac{3}{5}$$

$$\frac{\text{Volume of small cone}}{\text{Volume of large cone}} = \left(\frac{\text{Radius of small cone}}{\text{Radius of large cone}}\right)^3$$

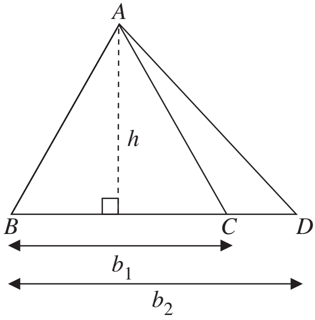
$$\frac{125}{\text{Volume of large cone}} = \left(\frac{3}{5}\right)^3$$

Volume of large cone = 579 cm^3 (to 3 s.f.)

15. If X and Y are two similar solids with the same density, then the ratio of their masses is given by $\frac{m_1}{m_2} = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{l_1}{l_2}\right)^3$.

Triangles Sharing the Same Height

16. If ΔABC and ΔABD share the same height h, then



$$\frac{\text{Area of }\Delta ABC}{\text{Area of }\Delta ABD} = \frac{\frac{1}{2}b_1h}{\frac{1}{2}b_2h} = \frac{b_1}{b_2}$$

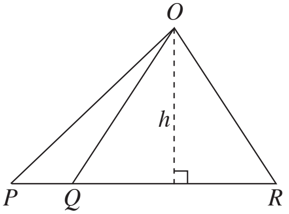
Example 5

In the diagram below, PQR is a straight line, $PQ = 2$ cm and $PR = 8$ cm. ΔOPQ and ΔOPR share the same height, h. Find the ratio of the area of ΔOPQ to the area of ΔOQR.

Solution

Both triangles share the same height, h.

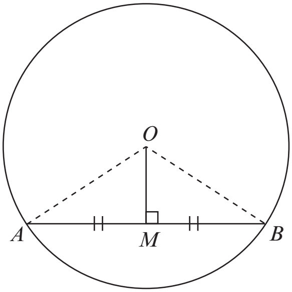
$$\frac{\text{Area of }\Delta OPQ}{\text{Area of }\Delta OQR} = \frac{\frac{1}{2}\times PQ\times h}{\frac{1}{2}\times QR\times h} = \frac{PQ}{QR} = \frac{2}{6} = \frac{1}{3}$$



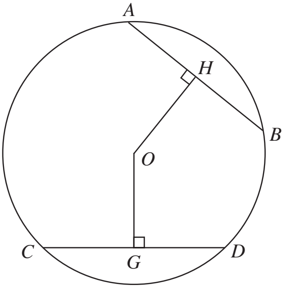
UNIT 2.3 Properties of Circles

Symmetric Properties of Circles

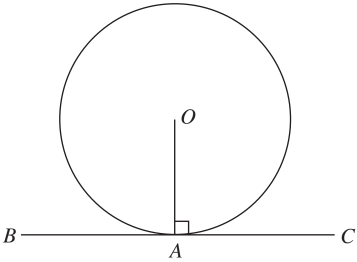
1. The perpendicular bisector of a chord AB of a circle passes through the centre of the circle, i.e. $AM = MB \Leftrightarrow OM \perp AB$.



2. If the chords AB and CD are equal in length, then they are equidistant from the centre, i.e. $AB = CD \Leftrightarrow OH = OG$.



3. The radius *OA* of a circle is perpendicular to the tangent at the point of contact, i.e. $O\hat{A}C = 90°$.

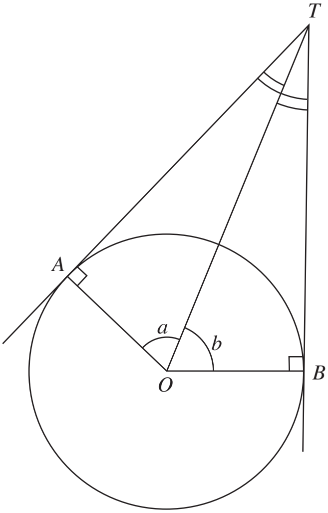


4. If *TA* and *TB* are tangents from *T* to a circle centre *O*, then

(i) $TA = TB$,

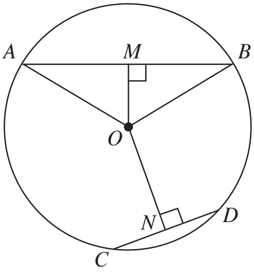
(ii) $\angle a = \angle b$,

(iii) *OT* bisects $A\hat{T}B$.



Example 1

In the diagram, O is the centre of the circle with chords AB and CD. $ON = 5$ cm and $CD = 5$ cm. $O\hat{C}D = 2O\hat{B}A$. Find the length of AB.



Solution

Radius = $OC = OD$

Radius = $\sqrt{5^2 + 2.5^2}$

$= 5.590$ cm (to 4 s.f.)

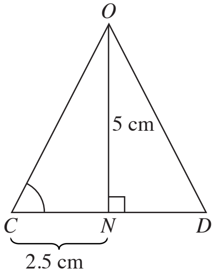
$\tan O\hat{C}D = \dfrac{ON}{AN}$

$= \dfrac{5}{2.5}$

$O\hat{C}D = 63.43°$ (to 2 d.p.)

$O\hat{B}A = 63.43° \div 2$

$= 31.72°$



OB = radius

$= 5.59$ cm

$\cos O\hat{B}A = \dfrac{BM}{OB}$

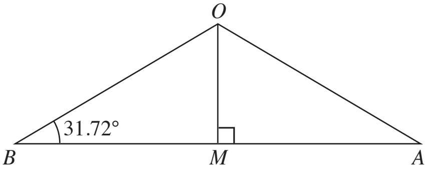
$\cos 31.72° = \dfrac{BM}{5.59}$

$BM = 5.59 \times \cos 31.72°$

$= 4.755$ cm (to 4 s.f.)

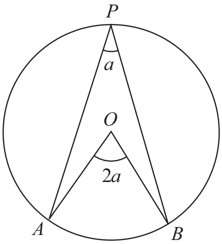
$AB = 2 \times 4.755$

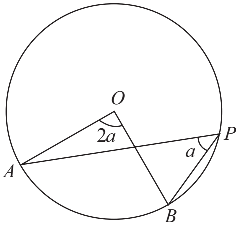
$= 9.51$ cm

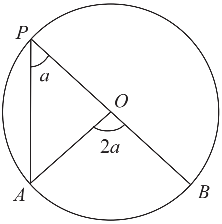


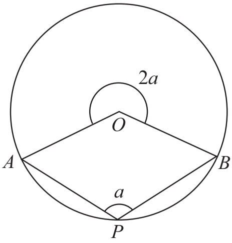
Angle Properties of Circles

5. An angle at the centre of a circle is twice that of any angle at the circumference subtended by the same arc, i.e. $\angle AOB = 2 \times \angle APB$.

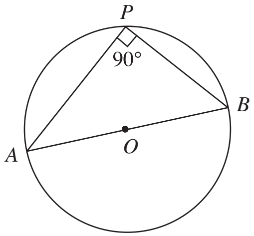




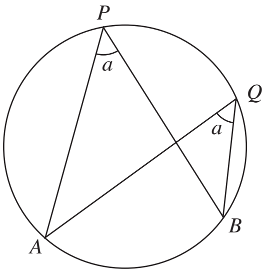




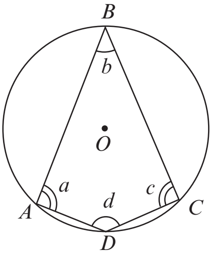
6. An angle in a semicircle is always equal to 90°, i.e. AOB is a diameter $\Leftrightarrow \angle APB = 90°$.



7. Angles in the same segment are equal, i.e. $\angle APB = \angle AQB$.

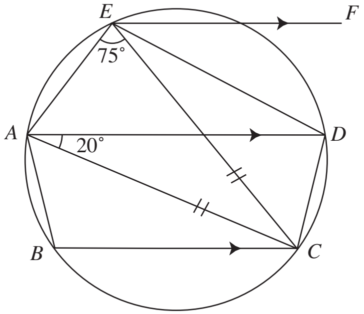


8. Angles in opposite segments are supplementary, i.e. $\angle a + \angle c = 180°$; $\angle b + \angle d = 180°$.



Example 2

In the diagram, A, B, C, D and E lie on a circle and $AC = EC$. The lines BC, AD and EF are parallel. $A\hat{E}C = 75°$ and $D\hat{A}C = 20°$.



Find

(i) $A\hat{C}B$, **(ii)** $A\hat{B}C$,

(iii) $B\hat{A}C$, **(iv)** $E\hat{A}D$,

(v) $F\hat{E}D$.

Solution

(i) $A\hat{C}B = D\hat{A}C$

$= 20°$ (alt. $\angle$s, BC // AD)

(ii) $A\hat{B}C + A\hat{E}C = 180°$ ($\angle$s in opp. segments)

$A\hat{B}C + 75° = 180°$

$A\hat{B}C = 180° - 75°$

$= 105°$

(iii) $B\hat{A}C = 180° - 20° - 105°$ ($\angle$ sum of Δ)

$= 55°$

(iv) $E\hat{A}C = A\hat{E}C = 75°$ (base $\angle$s of isos. Δ)

$E\hat{A}D = 75° - 20°$

$= 55°$

(v) $E\hat{C}A = 180° - 2 \times 75°$

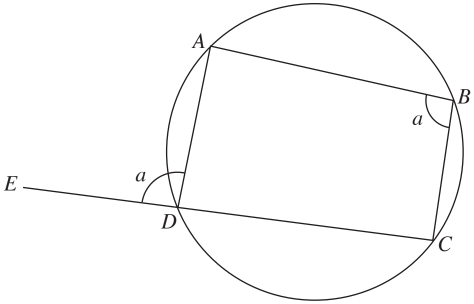
$= 30°$ ($\angle$ sum of Δ)

$E\hat{D}A = E\hat{C}A = 30°$ ($\angle$s in same segment)

$F\hat{E}D = E\hat{D}A = 30°$ (alt. $\angle$s, EF // AD)

9. The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle, i.e. $\angle ADC = 180° - \angle ABC$ ($\angle$s in opp. segments)

$\angle ADE = 180° - (180° - \angle ABC) = \angle ABC$

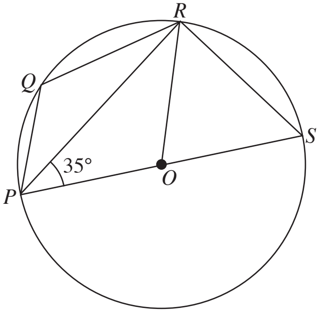


Example 3

In the diagram, O is the centre of the circle and $\angle RPS = 35°$. Find the following angles:

(a) $\angle ROS$,

(b) $\angle ORS$.



Solution

(a) $\angle ROS = 70°$ ($\angle$ at centre = 2 $\angle$ at circumference)

(b) $\angle OSR = 180° - 90° - 35°$ (rt. $\angle$ in a semicircle)

$= 55°$

$\angle ORS = 180° - 70° - 55°$ ($\angle$ sum of Δ)

$= 55°$

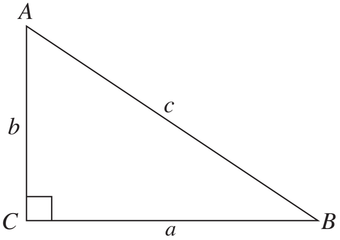
Alternatively,

$\angle ORS = \angle OSR$

$= 55°$ (base $\angle$s of isos. Δ)

UNIT 2.4 Pythagoras' Theorem and Trigonometry

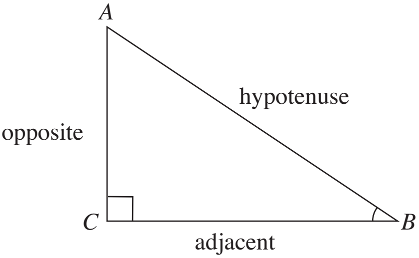
Pythagoras' Theorem



1. For a right-angled triangle ABC, if $\angle C = 90°$, then $AB^2 = BC^2 + AC^2$, i.e. $c^2 = a^2 + b^2$.

2. For a triangle ABC, if $AB^2 = BC^2 + AC^2$, then $\angle C = 90°$.

Trigonometric Ratios of Acute Angles



3. The side opposite the right angle C is called the hypotenuse. It is the longest side of a right-angled triangle.

4. In a triangle ABC, if $\angle C = 90°$,

then $\dfrac{AC}{AB} = \dfrac{\text{opp}}{\text{adj}}$ is called the sine of $\angle B$, or $\sin B = \dfrac{\text{opp}}{\text{hyp}}$,

$\dfrac{BC}{AB} = \dfrac{\text{adj}}{\text{hyp}}$ is called the cosine of $\angle B$, or $\cos B = \dfrac{\text{adj}}{\text{hyp}}$,

$\dfrac{AC}{BC} = \dfrac{\text{opp}}{\text{adj}}$ is called the tangent of $\angle B$, or $\tan B = \dfrac{\text{opp}}{\text{adj}}$.

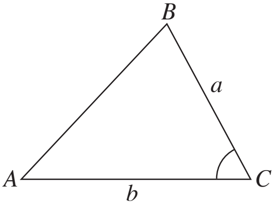
Trigonometric Ratios of Obtuse Angles

5. When θ is obtuse, i.e. $90° < \theta < 180°$,

$$\sin \theta = \sin (180° - \theta),$$
$$\cos \theta = -\cos (180° - \theta).$$
$$\tan \theta = -\tan (180° - \theta).$$

Area of a Triangle

6. Area of $\Delta ABC = \dfrac{1}{2}ab \sin C$



Sine Rule

7. In any ΔABC, the Sine Rule states that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

8. The Sine Rule can be used to solve a triangle if the following are given:

- two angles and the length of one side; or
- the lengths of two sides and one non-included angle.

Cosine Rule

9. In any ΔABC, the Cosine Rule states that

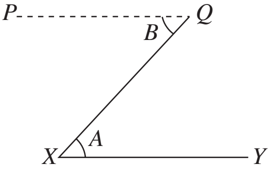
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

10. The Cosine Rule can be used to solve a triangle if the following are given:
- the lengths of all three sides; or
- the lengths of two sides and an included angle.

Angles of Elevation and Depression

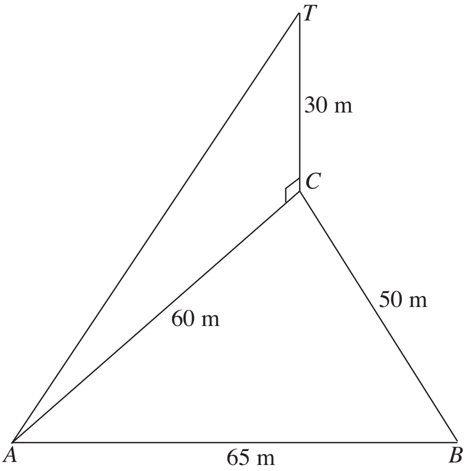


11. The angle A measured from the horizontal level XY is called the angle of elevation of Q from X.

12. The angle B measured from the horizontal level PQ is called the angle of depression of X from Q.

Example 1

In the figure, A, B and C lie on level ground such that $AB = 65$ m, $BC = 50$ m and $AC = 60$ m. T is vertically above C such that $TC = 30$ m.



Find

(i) $A\hat{C}B$,

(ii) the angle of elevation of T from A.

Solution

(i) Using cosine rule,

$$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos A\hat{C}B$$

$$65^2 = 60^2 + 50^2 - 2(60)(50)\cos A\hat{C}B$$

$$\cos A\hat{C}B = \frac{1875}{6000}$$

$$A\hat{C}B = 71.8° \text{ (to 1 d.p.)}$$

(ii) In ΔATC,

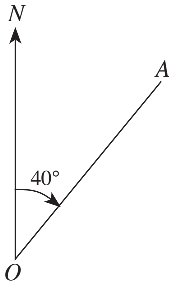
$$\tan T\hat{A}C = \frac{30}{60}$$

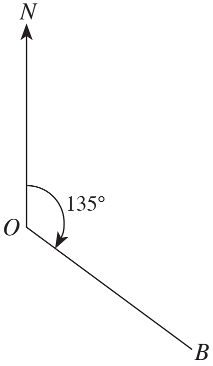
$$T\hat{A}C = 26.6° \text{ (to 1 d.p.)}$$

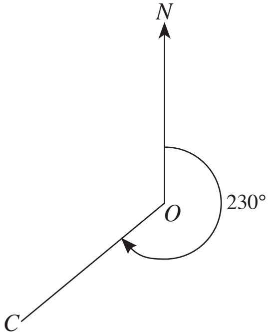
$\therefore$ Angle of elevation of T from A is 26.6°

Bearings

13. The bearing of a point A from another point O is an angle measured from the north, at O, in a clockwise direction and is written as a three-digit number.
e.g.



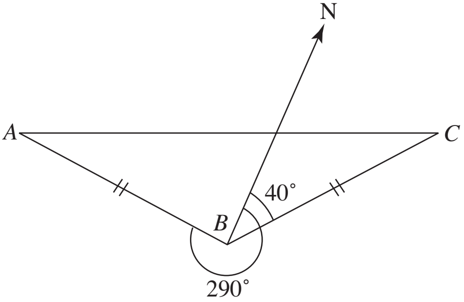




The bearing of A from O is 040°.
The bearing of B from O is 135°.
The bearing of C from O is 230°.

Example 2

The bearing of A from B is 290°. The bearing of C from B is 040°. $AB = BC$.

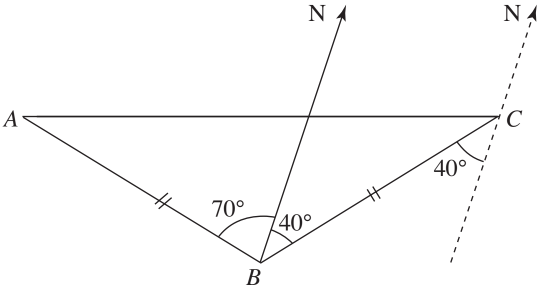


Find

(i) $B\hat{C}A$,

(ii) the bearing of A from C.

Solution



(i) $B\hat{C}A = \dfrac{180° - 70° - 40°}{2}$

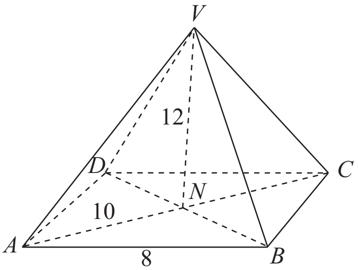
$= 35°$

(ii) Bearing of A from $C = 180° + 40° + 35°$

$= 255°$

14. The basic technique used to solve a three-dimensional problem is to reduce it to a problem in a plane.

Example 3



The figure shows a pyramid with a rectangular base, *ABCD*, and vertex *V*. The slant edges *VA*, *VB*, *VC* and *VD* are all equal in length and the diagonals of the base intersect at *N*. *AB* = 8 cm, *AC* = 10 cm and *VN* = 12 cm.

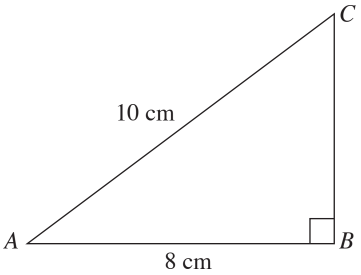
(i) Find the length of *BC*.

(ii) Find the length of *VC*.

(iii) Write down the tangent of the angle between *VN* and *VC*.

Solution

(i)



Using Pythagoras' Theorem,

$AC^2 = AB^2 + BC^2$

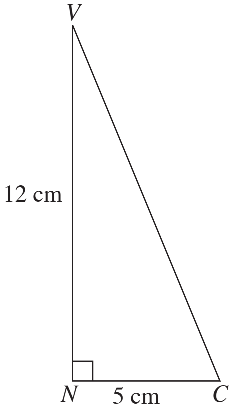
$10^2 = 8^2 + BC^2$

$BC^2 = 36$

$BC = 6$ cm

(ii) $CN = \frac{1}{2}AC$

$= 5$ cm



Using Pythagoras' Theorem,

$$\begin{aligned} VC^2 &= VN^2 + CN^2 \\ &= 12^2 + 5^2 \\ &= 169 \\ VC &= 13 \text{ cm} \end{aligned}$$

(iii) The angle between VN and VC is $C\hat{V}N$.

In ΔVNC,

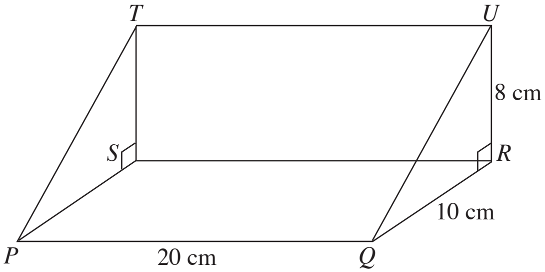
$$\begin{aligned} \tan C\hat{V}N &= \frac{CN}{VN} \\ &= \frac{5}{12} \end{aligned}$$

Example 4

The diagram shows a right-angled triangular prism.
Find

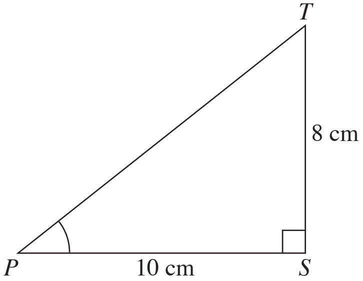
(a) $S\hat{P}T$,

(b) PU.



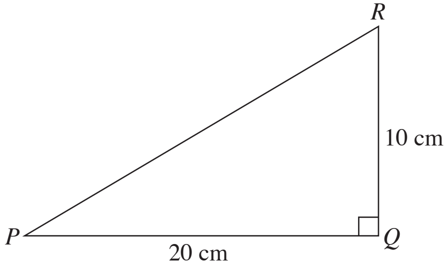
Solution

(a)

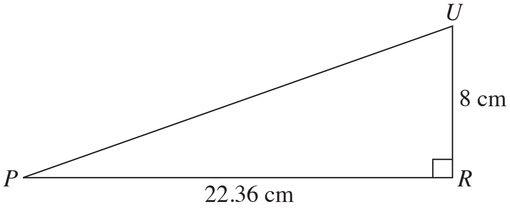


$$
\begin{aligned}
\tan S\hat{P}T &= \frac{ST}{PS} \\
&= \frac{8}{10} \\
S\hat{P}T &= 38.7° \text{ (to 1 d.p.)}
\end{aligned}
$$

(b)



$$\begin{aligned} PR^2 &= PQ^2 + QR^2 \\ &= 20^2 + 10^2 \\ &= 500 \\ PR &= 22.36 \text{ cm (to 4 s.f.)} \end{aligned}$$



$$\begin{aligned} PU^2 &= PR^2 + UR^2 \\ &= 22.36^2 + 8^2 \\ PU &= 23.7 \text{ cm (to 3 s.f.)} \end{aligned}$$

UNIT 2.5 Mensuration

Perimeter and Area of Figures

1.

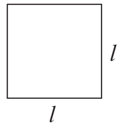
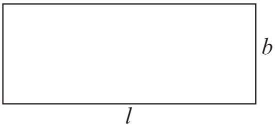
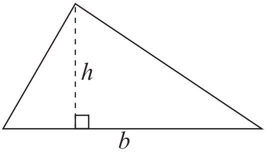
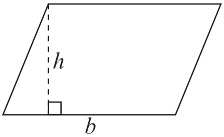
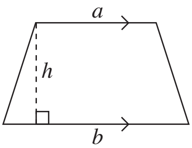
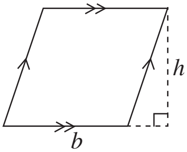
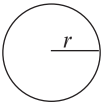
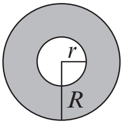
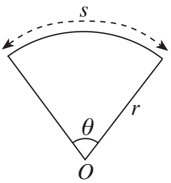
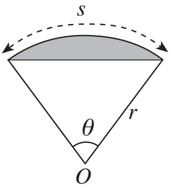
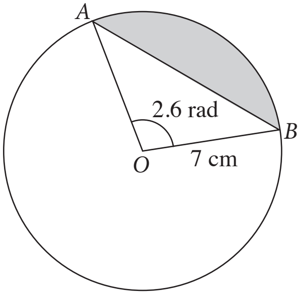
Figure	Diagram	Formulae
Square		Area = l^2 Perimeter = $4l$
Rectangle		Area = $l \times b$ Perimeter = $2(l + b)$
Triangle		Area = $\frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times b \times h$
Parallelogram		Area = base × height $= b \times h$
Trapezium		Area = $\frac{1}{2}(a + b)h$
Rhombus		Area = $b \times h$

Figure	**Diagram**	**Formulae**
Circle		Area = πr^2 Circumference = $2\pi r$
Annulus		Area = $\pi(R^2 - r^2)$
Sector		Arc length $s = \frac{\theta°}{360°} \times 2\pi r$ (where θ is in degrees) $= r\theta$ (where θ is in radians) Area = $\frac{\theta°}{360°} \times \pi r^2$ (where θ is in degrees) $= \frac{1}{2} r^2\theta$ (where θ is in radians)
Segment		Area = $\frac{1}{2} r^2(\theta - \sin \theta)$ (where θ is in radians)

Example 1

In the figure, O is the centre of the circle of radius 7 cm. AB is a chord and $\angle AOB = 2.6$ rad. The minor segment of the circle formed by the chord AB is shaded.



Find

(a) the length of the minor arc AB,

(b) the area of the shaded region.

Solution

(a) Length of minor arc $AB = 7(2.6)$

$$= 18.2 \text{ cm}$$

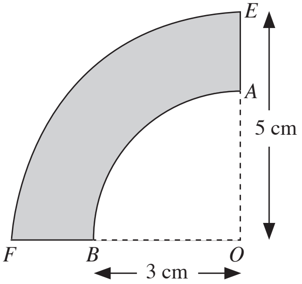
(b) Area of shaded region = Area of sector AOB – Area of ΔAOB

$$= \frac{1}{2}(7)^2(2.6) - \frac{1}{2}(7)^2 \sin 2.6$$

$$= 51.1 \text{ cm}^2 \text{ (to 3 s.f.)}$$

Example 2

In the figure, the radii of quadrants *ABO* and *EFO* are 3 cm and 5 cm respectively.



(a) Find the arc length of *AB*, in terms of π.

(b) Find the perimeter of the shaded region. Give your answer in the form $a + b\pi$.

Solution

(a) Arc length of *AB* = $\frac{3\pi}{2}$ cm

(b) Arc length *EF* = $\frac{5\pi}{2}$ cm

$$\text{Perimeter} = \frac{3\pi}{2} + \frac{5\pi}{2} + 2 + 2$$
$$= (4 + 4\pi)\text{ cm}$$

Degrees and Radians

2. π radians = 180°

$$1 \text{ radian} = \frac{180°}{\pi}$$

$$1° = \frac{\pi}{180°} \text{ radians}$$

3. To convert from degrees to radians and from radians to degrees:

$$\begin{tikzcd}
\text{Degree} \arrow[r, bend left, "\times\frac{\pi}{180°}"] & \text{Radian} \arrow[l, bend left, "\times\frac{180°}{\pi}"]
\end{tikzcd}$$

Conversion of Units

4. **Length**

1 m = 100 cm

1 cm = 10 mm

Area

$$\begin{aligned} 1 \text{ cm}^2 &= 10 \text{ mm} \times 10 \text{ mm} \\ &= 100 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} 1 \text{ m}^2 &= 100 \text{ cm} \times 100 \text{ cm} \\ &= 10\,000 \text{ cm}^2 \end{aligned}$$

Volume

$$\begin{aligned} 1 \text{ cm}^3 &= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} \\ &= 1000 \text{ mm}^3 \end{aligned}$$

$$\begin{aligned} 1 \text{ m}^3 &= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 1\,000\,000 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} 1 \text{ litre} &= 1000 \text{ ml} \\ &= 1000 \text{ cm}^3 \end{aligned}$$

Volume and Surface Area of Solids

5.

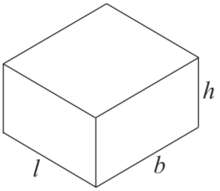
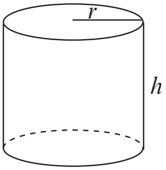
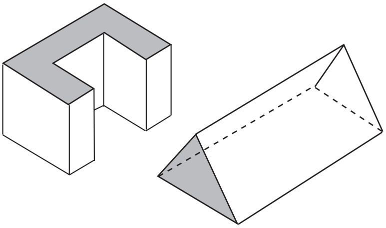
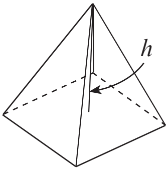
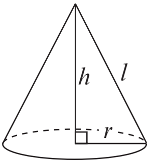
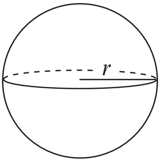
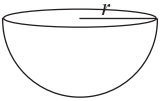
Figure	**Diagram**	**Formulae**
Cuboid		Volume = $l \times b \times h$ Total surface area = $2(lb + lh + bh)$
Cylinder		Volume = $\pi r^2 h$ Curved surface area = $2\pi rh$ Total surface area = $2\pi rh + 2\pi r^2$
Prism		Volume = Area of cross section × length Total surface area = Perimeter of the base × height + 2(base area)
Pyramid		Volume = $\frac{1}{3}$ × base area × h
Cone		Volume = $\frac{1}{3}\pi r^2 h$ Curved surface area = πrl (where l is the slant height) Total surface area = $\pi rl + \pi r^2$

Figure	Diagram	Formulae
Sphere		Volume = $\frac{4}{3}\pi r^3$ Surface area = $4\pi r^2$
Hemisphere		Volume = $\frac{2}{3}\pi r^3$ Surface area = $2\pi r^2 + \pi r^2$ $= 3\pi r^2$

Example 3

(a) A sphere has a radius of 10 cm. Calculate the volume of the sphere.

(b) A cuboid has the same volume as the sphere in part **(a)**. The length and breadth of the cuboid are both 5 cm. Calculate the height of the cuboid.

Leave your answers in terms of π.

Solution

(a) Volume $= \frac{4\pi(10)^3}{3}$

$= \frac{4000\,\pi}{3}$ cm^3

(b) Volume of cuboid $= l \times b \times h$

$\frac{4000\,\pi}{3} = 5 \times 5 \times h$

$h = \frac{160\pi}{3}$ cm

Example 4

The diagram shows a solid which consists of a pyramid with a square base attached to a cuboid. The vertex V of the pyramid is vertically above M and N, the centres of the squares $PQRS$ and $ABCD$ respectively. $AB = 30$ cm, $AP = 40$ cm and $VN = 20$ cm.

(a) Find

(i) VA,

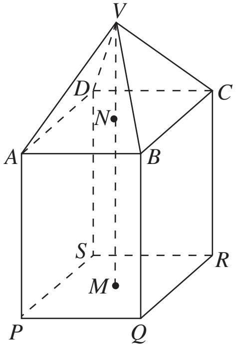
(ii) $V\hat{A}C$.

(b) Calculate

(i) the volume,

(ii) the total surface area

of the solid.



Solution

(a) **(i)** Using Pythagoras' Theorem,

$$AC^2 = AB^2 + BC^2$$
$$= 30^2 + 30^2$$
$$= 1800$$
$$AC = \sqrt{1800} \text{ cm}$$
$$AN = \frac{1}{2}\sqrt{1800} \text{ cm}$$

Using Pythagoras' Theorem,

$$VA^2 = VN^2 + AN^2$$
$$= 20^2 + \left(\frac{1}{2}\sqrt{1800}\right)^2$$
$$= 850$$
$$VA = \sqrt{850}$$
$$= 29.2 \text{ cm (to 3 s.f.)}$$

(ii) $V\hat{A}C = V\hat{A}N$

In ΔVAN,

$$\tan V\hat{A}N = \frac{VN}{AN}$$

$$= \frac{20}{\frac{1}{2}\sqrt{1800}}$$

$$= 0.9428 \text{ (to 4 s.f.)}$$

$$V\hat{A}N = 43.3° \text{ (to 1 d.p.)}$$

(b) (i) Volume of solid = Volume of cuboid + Volume of pyramid

$$= (30)(30)(40) + \frac{1}{3}(30)^2(20)$$

$$= 42\,000 \text{ cm}^3$$

(ii) Let X be the midpoint of AB.

Using Pythagoras' Theorem,

$VA^2 = AX^2 + VX^2$

$850 = 15^2 + VX^2$

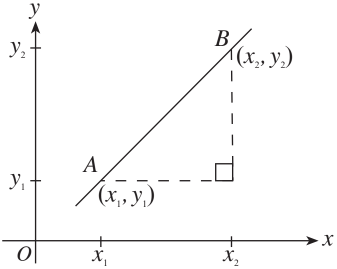
$VX^2 = 625$

$VX = 25$ cm

Total surface area $= 30^2 + 4(40)(30) + 4\left(\frac{1}{2}\right)(30)(25)$

$= 7200 \text{ cm}^2$

UNIT 2.6 Coordinate Geometry



Gradient

1. The gradient of the line joining any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{y_1 - y_2}{x_1 - x_2}$.

2. Parallel lines have the same gradient.

Distance

3. The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\,.$$

Example 1

The gradient of the line joining the points $A(x, 9)$ and $B(2, 8)$ is $\frac{1}{2}$.

(a) Find the value of x.

(b) Find the length of AB.

Solution

(a) Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$$\frac{8 - 9}{2 - x} = \frac{1}{2}$$

$$-2 = 2 - x$$

$$x = 4$$

(b) Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(2 - 4)^2 + (8 - 9)^2}$$

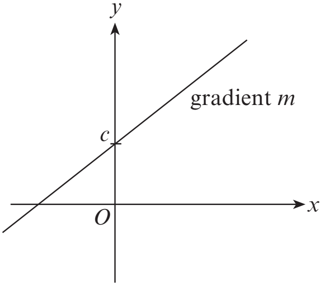
$$= \sqrt{(-2)^2 + (-1)^2}$$

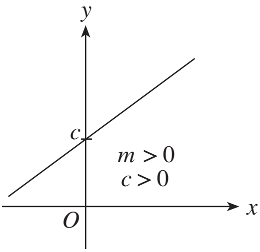
$$= \sqrt{5}$$

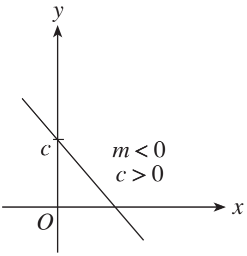
$$= 2.24 \text{ (to 3 s.f.)}$$

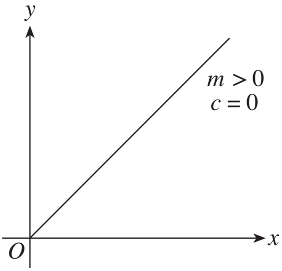
Equation of a Straight Line

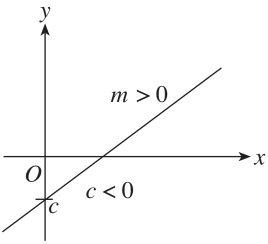
4. The equation of the straight line with gradient m and y-intercept c is $y = mx + c$.

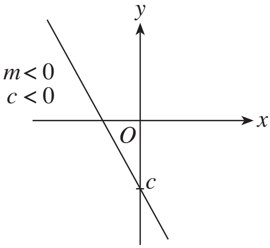


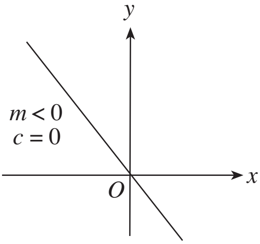

y
c
m > 0
c > 0
O
x

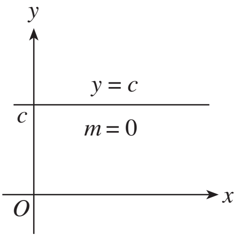

y
c
m < 0
c > 0
O
x

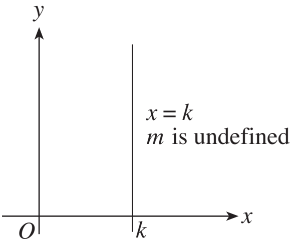

y
m > 0
c = 0
O
x


y
m > 0
O
x
c < 0
c


y
m < 0
c < 0
x
O
c


y
m < 0
c = 0
x
O


y
y = c
c
m = 0
O
x


y
x = k
m is undefined
O
k
x

Example 2

A line passes through the points $A(6, 2)$ and $B(5, 5)$. Find the equation of the line.

Solution

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 2}{5 - 6}$$
$$= -3$$

Equation of line: $y = mx + c$

$y = -3x + c$

To find c, we substitute $x = 6$ and $y = 2$ into the equation above.

(We can find c by substituting the coordinates of any point that lies on the line into the equation)

$2 = -3(6) + c$

$c = 20$

$\therefore$ Equation of line: $y = -3x + 20$

5. The equation of a horizontal line is of the form $y = c$.

6. The equation of a vertical line is of the form $x = k$.

Example 3

The points A, B and C are $(8, 7)$, $(11, 3)$ and $(3, –3)$ respectively.

(a) Find the equation of the line parallel to AB and passing through C.

(b) Show that AB is perpendicular to BC.

(c) Calculate the area of triangle ABC.

Solution

(a) Gradient of $AB = \dfrac{3-7}{11-8}$

$$= -\frac{4}{3}$$

$$y = -\frac{4}{3}x + c$$

Substitute $x = 3$ and $y = –3$:

$$–3 = -\frac{4}{3}(3) + c$$

$$–3 = –4 + c$$

$$c = 1$$

$\therefore$ Equation of line: $y\ -\dfrac{4}{3}x + 1$

(b) $AB = \sqrt{(11-8)^2 + (3-7)^2}$

$$= 5 \text{ units}$$

$$BC = \sqrt{(3-11)^2 + (-3-3)^2}$$

$$= 10 \text{ units}$$

$$AC = \sqrt{(3-8)^2 + (-3-7)^2}$$

$$= \sqrt{125} \text{ units}$$

Since $AB^2 + BC^2 = 5^2 + 10^2$

$$= 125$$

$$= AC^2,$$

Pythagoras' Theorem can be applied.

$\therefore$ AB is perpendicular to BC.

(c) Area of $\Delta ABC = \dfrac{1}{2}(5)(10)$

$$= 25 \text{ units}^2$$

UNIT 2.7

Vectors in Two Dimensions

(not included for NA)

Vectors

1. A vector has both magnitude and direction but a scalar has magnitude only.

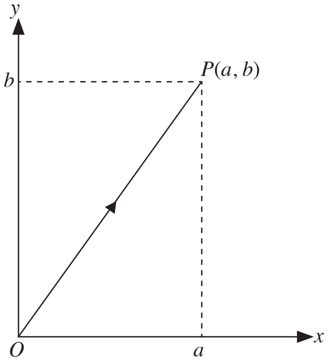
2. A vector may be represented by $\overrightarrow{OA}$, $\underset{\sim}{a}$ or **a**.

Magnitude of a Vector

3. The magnitude of a column vector $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ is given by $|\mathbf{a}| = \sqrt{x^2 + y^2}$.

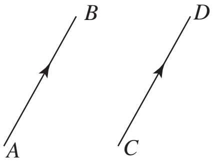
Position Vectors

4. If the point P has coordinates (a, b), then the position vector of P, $\overrightarrow{OP}$, is written as $\overrightarrow{OP} = \begin{pmatrix} a \\ b \end{pmatrix}$.



Equal Vectors

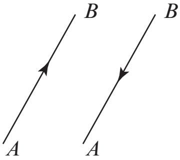
5. Two vectors are equal when they have the same direction and magnitude.



$\overrightarrow{AB} = \overrightarrow{CD}$

Negative Vector

6. $\overrightarrow{BA}$ is the negative of $\overrightarrow{AB}$.
$\overrightarrow{BA}$ is a vector having the same magnitude as $\overrightarrow{AB}$ but having direction opposite to that of $\overrightarrow{AB}$.
We can write $\overrightarrow{BA} = -\overrightarrow{AB}$ and $\overrightarrow{AB} = -\overrightarrow{BA}$.



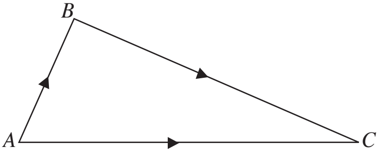
Zero Vector

7. A vector whose magnitude is zero is called a zero vector and is denoted by **0**.

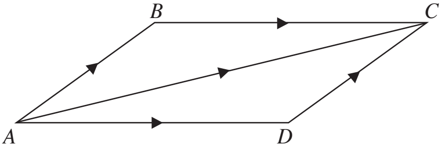
Sum and Difference of Two Vectors

8. The sum of two vectors, **a** and **b**, can be determined by using the Triangle Law or Parallelogram Law of Vector Addition.

9. Triangle law of addition: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

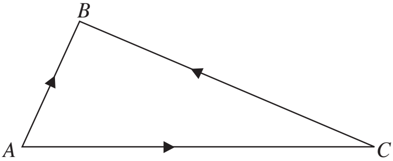


10. Parallelogram law of addition: $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



11. The difference of two vectors, **a** and **b**, can be determined by using the Triangle Law of Vector Subtraction.

$\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$



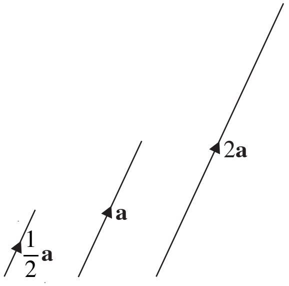
12. For any two column vectors $\mathbf{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} r \\ s \end{pmatrix}$,

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p+r \\ q+s \end{pmatrix}$$

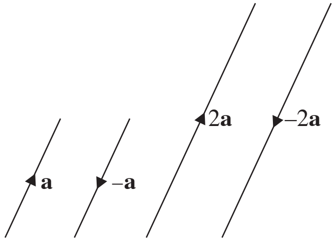
and $\mathbf{a} - \mathbf{b} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p-r \\ q-s \end{pmatrix}$.

Scalar Multiple

13. When $k > 0$, $k\mathbf{a}$ is a vector having the same direction as that of $\mathbf{a}$ and magnitude equal to k times that of $\mathbf{a}$.

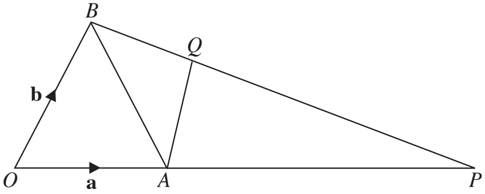


14. When $k < 0$, $k\mathbf{a}$ is a vector having a direction opposite to that of $\mathbf{a}$ and magnitude equal to k times that of $\mathbf{a}$.



Example 1

In $\Delta OAB, \overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
O, A and P lie in a straight line, such that $OP = 3OA$.
Q is the point on BP such that $4BQ = PB$.



Express in terms of **a** and **b**,

(i) $\overrightarrow{BP}$,

(ii) $\overrightarrow{QB}$.

Solution

(i) $\overrightarrow{BP} = -\mathbf{b} + 3\mathbf{a}$

(ii) $\overrightarrow{QB} = \frac{1}{4}\overrightarrow{PB}$

$= \frac{1}{4}(-3\mathbf{a} + \mathbf{b})$

Parallel Vectors

15. If $\mathbf{a} = k\mathbf{b}$, where k is a scalar and $k \neq 0$, then **a** is parallel to **b** and $|\mathbf{a}| = k|\mathbf{b}|$.

16. If **a** is parallel to **b**, then $\mathbf{a} = k\mathbf{b}$, where k is a scalar and $k \neq 0$.

Example 2

Given that $\begin{pmatrix} -15 \\ 9 \end{pmatrix}$ and $\begin{pmatrix} w \\ -3 \end{pmatrix}$ are parallel vectors, find the value of w.

Solution

Since $\begin{pmatrix} -15 \\ 9 \end{pmatrix}$ and $\begin{pmatrix} w \\ -3 \end{pmatrix}$ are parallel,

let $\begin{pmatrix} -15 \\ 9 \end{pmatrix} = k\begin{pmatrix} w \\ -3 \end{pmatrix}$, where k is a scalar.

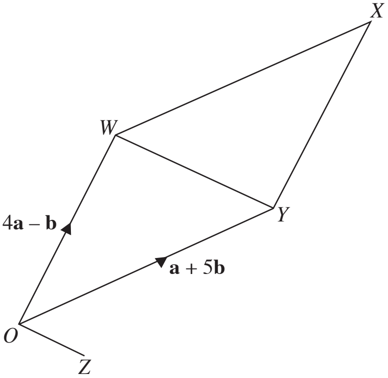
$$9 = -3k$$
$$k = -3$$

i.e. $-15 = -3w$

$$w = 5$$

Example 3

Figure $WXYO$ is a parallelogram.



(a) Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{XY}$,

(ii) $\overrightarrow{WY}$.

(b) Z is the point such that $\overrightarrow{OZ} = -\mathbf{a} + 2\mathbf{b}$.

- **(i)** Determine if $\overrightarrow{WY}$ is parallel to $\overrightarrow{OZ}$.
- **(ii)** Given that the area of triangle OWY is 36 units^2, find the area of triangle OYZ.

Solution

(a) **(i)**
$$\begin{aligned}\overrightarrow{XY} &= -\overrightarrow{OW}\\ &= \mathbf{b} - 4\mathbf{a}\end{aligned}$$

(ii)
$$\begin{aligned}\overrightarrow{WY} &= \overrightarrow{WO} + \overrightarrow{OY}\\ &= \mathbf{b} - 4\mathbf{a} + \mathbf{a} + 5\mathbf{b}\\ &= -3\mathbf{a} + 6\mathbf{b}\end{aligned}$$

(b) **(i)**
$$\begin{aligned}\overrightarrow{OZ} &= -\mathbf{a} + 2\mathbf{b}\\ \overrightarrow{WY} &= -3\mathbf{a} + 6\mathbf{b}\\ &= 3(-\mathbf{a} + 2\mathbf{b})\end{aligned}$$

Since $\overrightarrow{WY} = 3\overrightarrow{OZ}$, $\overrightarrow{WY}$ is parallel to $\overrightarrow{OZ}$.

(ii) ΔOYZ and ΔOWY share a common height h.

$$\begin{aligned}\frac{\text{Area of }\Delta OYZ}{\text{Area of }\Delta OWY} &= \frac{\frac{1}{2}\times OZ\times h}{\frac{1}{2}\times WY\times h}\\ &= \frac{\left|\overrightarrow{OZ}\right|}{\left|\overrightarrow{WY}\right|}\\ &= \frac{1}{3}\\ \frac{\text{Area of }\Delta OYZ}{36} &= \frac{1}{3}\end{aligned}$$

$\therefore$ Area of $\Delta OYZ = 12$ units^2

17. If $m\mathbf{a} + n\mathbf{b} = h\mathbf{a} + k\mathbf{b}$, where m, n, h and k are scalars and $\mathbf{a}$ is parallel to $\mathbf{b}$, then $m = h$ and $n = k$.

Collinear Points

18. If the points A, B and C are such that $\overrightarrow{AB} = k\overrightarrow{BC}$, then A, B and C are collinear, i.e. A, B and C lie on the same straight line.

19. To prove that 3 points A, B and C are collinear, we need to show that:

$\overrightarrow{AB} = k\,\overrightarrow{BC}$ or $\overrightarrow{AB} = k\,\overrightarrow{AC}$ or $\overrightarrow{AC} = k\,\overrightarrow{BC}$

Example 4

Show that A, B and C lie in a straight line.

$\overrightarrow{OA} = p$ $\quad\overrightarrow{OB} = q$ $\quad\overrightarrow{BC} = \frac{1}{3}p - \frac{1}{3}q$

Solution

$$\begin{aligned}\overrightarrow{AB} &= q - p \\ \overrightarrow{BC} &= \frac{1}{3}p - \frac{1}{3}q \\ &= -\frac{1}{3}(q - p) \\ &= -\frac{1}{3}\overrightarrow{AB}\end{aligned}$$

Thus $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$. Hence, the three points lie in a straight line.

UNIT

3.1

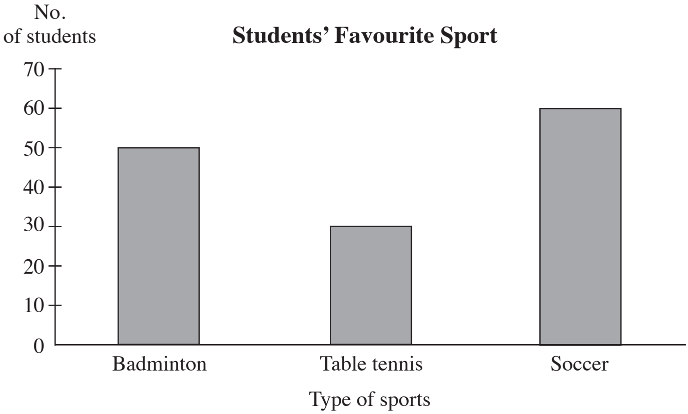
Data Handling

Bar Graph

1. In a bar graph, each bar is drawn having the same width and the length of each bar is proportional to the given data.

2. An advantage of a bar graph is that the data sets with the lowest frequency and the highest frequency can be easily identified.

e.g.



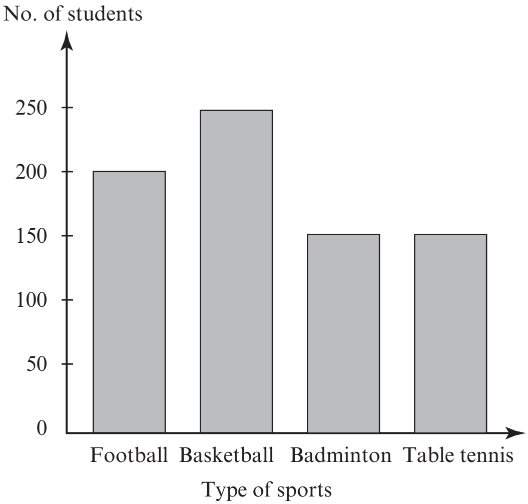
Example 1

The table shows the number of students who play each of the four types of sports.

Type of sports	No. of students
Football	200
Basketball	250
Badminton	150
Table tennis	150
Total	**750**

Represent the information in a bar graph.

Solution



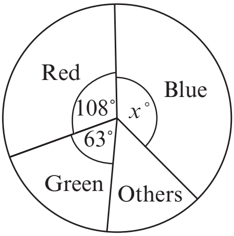
Pie Chart

3. A pie chart is a circle divided into several sectors and the angles of the sectors are proportional to the given data.

4. An advantage of a pie chart is that the size of each data set in proportion to the entire set of data can be easily observed.

Example 2

Each member of a class of 45 boys was asked to name his favourite colour. Their choices are represented on a pie chart.



(i) If 15 boys said they liked blue, find the value of x.
(ii) Find the percentage of the class who said they liked red.

Solution

(i) $x = \frac{15}{45} \times 360$

$= 120$

(ii) Percentage of the class who said they liked red $= \frac{108°}{360°} \times 100\%$

$= 30\%$

Histogram

5. A histogram is a vertical bar chart with no spaces between the bars (or rectangles). The frequency corresponding to a class is represented by the area of a bar whose base is the class interval.

6. An advantage of using a histogram is that the data sets with the lowest frequency and the highest frequency can be easily identified.

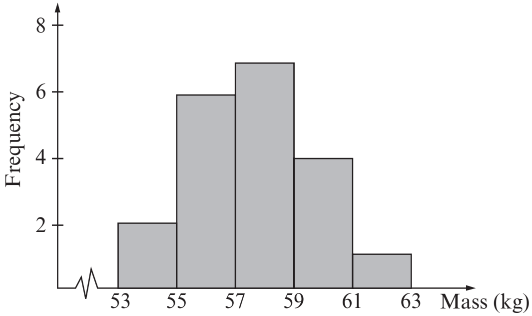
Example 3

The table shows the masses of the students in the school's track team.

Mass (m) in kg	**Frequency**
$53 < m \leqslant 55$	2
$55 < m \leqslant 57$	6
$57 < m \leqslant 59$	7
$59 < m \leqslant 61$	4
$61 < m \leqslant 63$	1

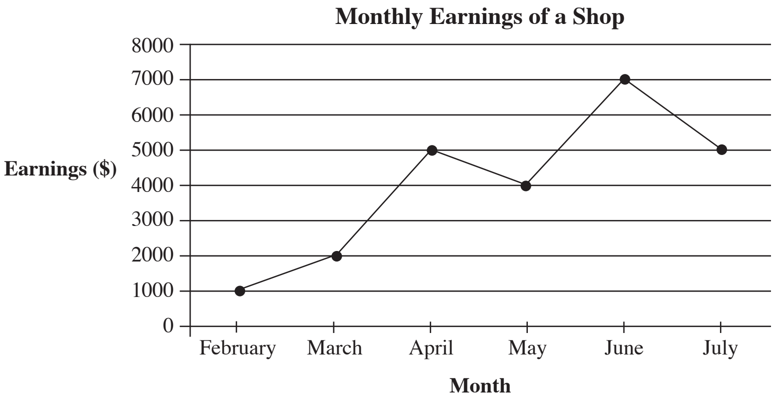
Represent the information on a histogram.

Solution



Line Graph

7. A line graph usually represents data that changes with time. Hence, the horizontal axis usually represents a time scale (e.g. hours, days, years).
e.g.



UNIT 3.2

Data Analysis

Dot Diagram

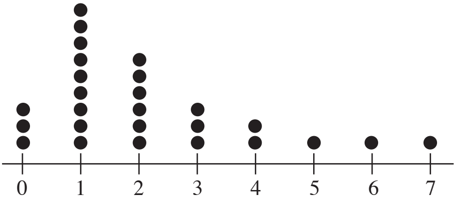
1. A dot diagram consists of a horizontal number line and dots placed above the number line, representing the values in a set of data.

Example 1

The table shows the number of goals scored by a soccer team during the tournament season.

Number of goals	0	1	2	3	4	5	6	7
Number of matches	3	9	6	3	2	1	1	1

The data can be represented on a dot diagram.



Example 2

The marks scored by twenty students in a placement test are as follows:

42	42	49	31	34	42	40	43	35	38
34	35	39	45	42	42	35	45	40	41

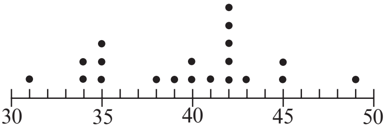
(a) Illustrate the information on a dot diagram.

(b) Write down

- **(i)** the lowest score,
- **(ii)** the highest score,
- **(iii)** the modal score.

Solution

(a)



(b) **(i)** Lowest score = 31

- **(ii)** Highest score = 49
- **(iii)** Modal score = 42

2. An advantage of a dot diagram is that it is an easy way to display small sets of data which do not contain many distinct values.

Stem-and-Leaf Diagram

3. In a stem-and-leaf diagram, the stems must be arranged in numerical order and the leaves must be arranged in ascending order.

4. An advantage of a stem-and-leaf diagram is that the individual data values are retained.

e.g. The ages of 15 shoppers are as follows:

32	34	13	29	38
36	14	28	37	13
42	24	20	11	25

The data can be represented on a stem-and-leaf diagram.

Stem	**Leaf**
1	1 3 3 4
2	0 4 5 8 9
3	2 4 6 7 8
4	2

Key: 1 | 3 means 13 years old
The tens are represented as stems and the ones are represented as leaves.
The values of the stems and the leaves are arranged in ascending order.

Stem-and-Leaf Diagram with Split Stems

5. If a stem-and-leaf diagram has more leaves on some stems, we can break each stem into two halves.

e.g. The stem-and-leaf diagram represents the number of customers in a store.

Stem	**Leaf**
4	0 3 5 8
5	1 3 3 4 5 6 8 8 9
6	2 5 7

Key: 4 | 0 means 40 customers

The information can be shown as a stem-and-leaf diagram with split stems.

Stem	**Leaf**
4	0 3 5 8
5	1 3 3 4
5	5 6 8 8 9
6	2 5 7

Key: 4 | 0 means 40 customers

Back-to-Back Stem-and-Leaf Diagram

6. If we have two sets of data, we can use a back-to-back stem-and-leaf diagram with a common stem to represent the data.

e.g. The scores for a quiz of two classes are shown in the table.

Class *A*	55	98	67	84	85	92	75	78	89	64
	72	60	86	91	97	58	63	86	92	74
Class *B*	56	67	92	50	64	83	84	67	90	83
	68	75	81	93	99	76	87	80	64	58

A back-to-back stem-and-leaf diagram can be constructed based on the given data.

Leaves for Class *B*	**Stem**	**Leaves for Class *A***
8 6 0	5	5 8
8 7 7 4 4	6	0 3 4 7
6 5	7	2 4 5 8
7 4 3 3 1 0	8	4 5 6 6 9
9 3 2 0	9	1 2 2 7 8

Key: 58 means 58 marks

Note that the leaves for Class *B* are arranged in ascending order from the right to the left.

Measures of Central Tendency

7. The three common measures of central tendency are the mean, median and mode.

Mean

8. The mean of a set of n numbers $x_1, x_2, x_3, \ldots, x_n$ is denoted by $\bar{x}$.

9. For ungrouped data,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum x}{n}.$$

10. For grouped data,

$$\bar{x} = \frac{\sum fx}{\sum f}$$

where f is the frequency of data in each class interval and x is the mid-value of the interval.

Median

11. The median is the value of the middle term of a set of numbers arranged in ascending order.

Given a set of n terms, if n is odd, the median is the $\left(\frac{n+1}{2}\right)^{\text{th}}$ term;

if n is even, the median is the average of the two middle terms.

e.g. Given the set of data: $5, 6, \overset{\downarrow}{7}, 8, 9$.
There is an odd number of data.
Hence, median is 7.

e.g. Given a set of data $5, 6, \overset{\downarrow}{}7, 8$.
There is an even number of data.
Hence, median is 6.5.

Example 3

The table records the number of mistakes made by 60 students during an exam.

Number of students	24	x	13	y	5
Number of mistakes	5	6	7	8	9

(a) Show that $x + y = 18$.
(b) Find an equation of the mean, given that the mean number of mistakes made is 6.3. Hence, find the values of x and y.
(c) State the median number of mistakes made.

Solution

(a) Since there are 60 students in total,

$$24 + x + 13 + y + 5 = 60$$
$$x + y + 42 = 60$$
$$x + y = 18$$

(b) Since the mean number of mistakes made is 6.3,

$$\text{Mean} = \frac{\text{Total number of mistakes made by 60 students}}{\text{Number of students}}$$

$$6.3 = \frac{24(5)+6x+13(7)+8y+5(9)}{60}$$

$6.3(60) = 120 + 6x + 91 + 8y + 45$

$378 = 256 + 6x + 8y$

$6x + 8y = 122$

$3x + 4y = 61$

To find the values of x and y, solve the pair of simultaneous equations obtained above.

$x + y = 18$ —— (1)

$3x + 4y = 61$ —— (2)

$3 \times (1)$:

$3x + 3y = 54$ —— (3)

$(2) - (3)$: $y = 7$

When $y = 7$, $x = 11$.

(c) Since there are 60 students, the 30th and 31st students are in the middle. The 30th and 31st students make 6 mistakes each. Therefore, the median number of mistakes made is 6.

Mode

12. The mode of a set of numbers is the number with the highest frequency.

13. If a set of data has two values which occur the most number of times, we say that the distribution is bimodal.

e.g. Given a set of data: 5, 6, 6, 6, 7, 7, 8, 8, 9.
6 occurs the most number of times.
Hence, the mode is 6.

Standard Deviation

14. The standard deviation, s, measures the spread of a set of data from its mean.

15. For ungrouped data,

$$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \quad \text{or} \quad s = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

16. For grouped data,

$$s = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} \quad \text{or} \quad s = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Example 4

The following set of data shows the number of books borrowed by 20 children during their visit to the library.

0, 2, 4, 3, 1, 1, 2, 0, 3, 1
1, 2, 1, 1, 2, 3, 2, 2, 1, 2

Calculate the standard deviation.

Solution

Represent the set of data in the table below.

Number of books borrowed	0	1	2	3	4
Number of children	2	7	7	3	1

Standard deviation

$$= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{0^2(2)+1^2(7)+2^2(7)+3^2(3)+4^2(1)}{20} - \left(\frac{0(2)+1(7)+2(7)+3(3)+4(1)}{20}\right)^2}$$

$$= \sqrt{\frac{78}{20} - \left(\frac{34}{20}\right)^2}$$

= 1.00 (to 3 s.f.)

Example 5

The mass, in grams, of 80 stones are given in the table.

Mass (m) in grams	**Frequency**
$20 < m \leqslant 30$	20
$30 < m \leqslant 40$	30
$40 < m \leqslant 50$	20
$50 < m \leqslant 60$	10

Find the mean and the standard deviation of the above distribution.

Solution

Mid-value (x)	**Frequency (f)**	fx	fx^2
25	20	500	12 500
35	30	1050	36 750
45	20	900	40 500
55	10	550	30 250

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$= \frac{500 + 1050 + 900 + 550}{20 + 30 + 20 + 10}$$

$$= \frac{3000}{80}$$

$$= 37.5 \text{ g}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

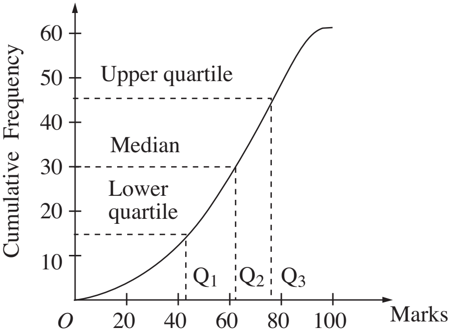
$$= \sqrt{\frac{12\,500 + 36\,750 + 40\,500 + 30\,250}{80} - \left(\frac{3000}{80}\right)^2}$$

$$= \sqrt{1500 - 37.5^2}$$

$$= 9.68 \text{ g (to 3 s.f.)}$$

Cumulative Frequency Curve

17. The following figure shows a cumulative frequency curve.



18. Q_1 is called the lower quartile or the 25th percentile.

19. Q_2 is called the median or the 50th percentile.

20. Q_3 is called the upper quartile or the 75th percentile.

21. $Q_3 - Q_1$ is called the interquartile range.

Example 6

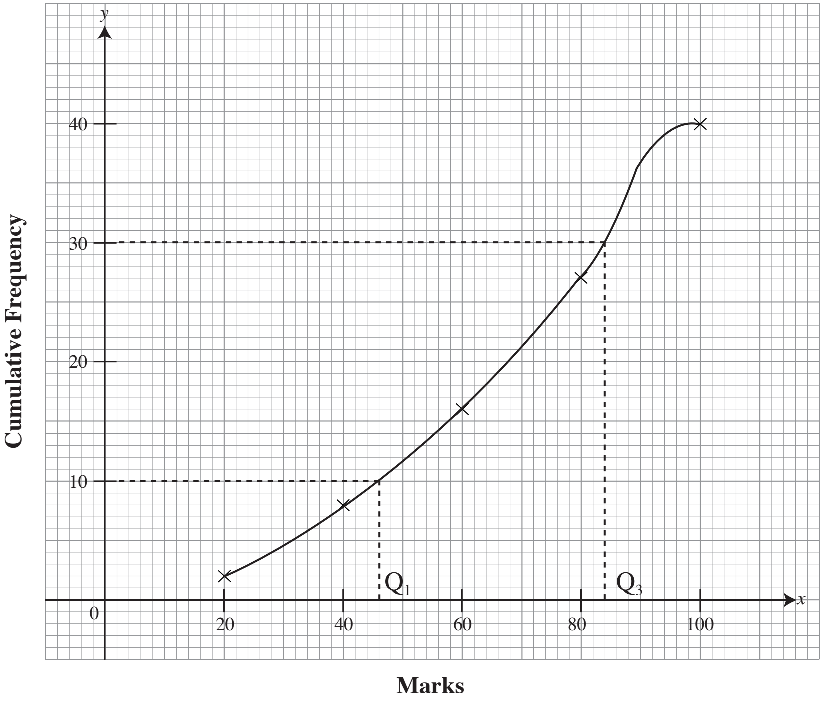
The exam results of 40 students were recorded in the frequency table below.

Mass (m) in grams	**Frequency**
$0 < m \leqslant 20$	2
$20 < m \leqslant 40$	4
$40 < m \leqslant 60$	8
$60 < m \leqslant 80$	14
$80 < m \leqslant 100$	12

Construct a cumulative frequency table and the draw a cumulative frequency curve. Hence, find the interquartile range.

Solution

Mass (m) in grams	**Cumulative Frequency**
$x \leqslant 20$	2
$x \leqslant 40$	6
$x \leqslant 60$	14
$x \leqslant 80$	28
$x \leqslant 100$	40



Lower quartile = 46

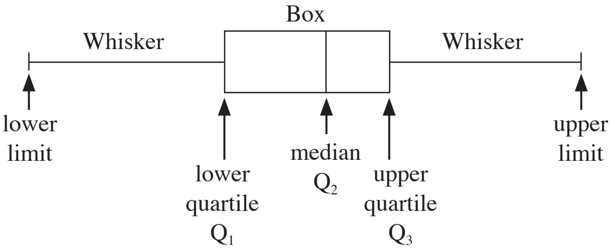
Upper quartile = 84

Interquartile range = 84 – 46

= 38

Box-and-Whisker Plot

22. The following figure shows a box-and-whisker plot.



23. A box-and-whisker plot illustrates the range, the quartiles and the median of a frequency distribution.

UNIT 3.3 Probability

1. Probability is a measure of chance.

2. A sample space is the collection of all the possible outcomes of a probability experiment.

Example 1

A fair six-sided die is rolled. Write down the sample space and state the total number of possible outcomes.

Solution

A die has the numbers 1, 2, 3, 4, 5 and 6 on its faces,
i.e. the sample space consists of the numbers 1, 2, 3, 4, 5 and 6.

Total number of possible outcomes = 6

3. In a probability experiment with m equally likely outcomes, if k of these outcomes favour the occurrence of an event E, then the probability, P(E), of the event happening is given by

$$P(E) = \frac{\text{Number of favourable outcomes for event } E}{\text{Total number of possible outcomes}} = \frac{k}{m}.$$

Example 2

A card is drawn at random from a standard pack of 52 playing cards.
Find the probability of drawing

(i) a King,

(ii) a spade.

Solution

Total number of possible outcomes = 52

(i) $\text{P(drawing a King)} = \frac{4}{52}$ (There are 4 Kings in a deck.)

$$= \frac{1}{13}$$

(ii) $\text{P(drawing a Spade)} = \frac{13}{52}$ (There are 13 spades in a deck.)

Properties of Probability

4. For any event E, $0 \leqslant \mathrm{P}(E) \leqslant 1$.

5. If E is an impossible event, then $\mathrm{P}(E) = 0$, i.e. it will never occur.

6. If E is a certain event, then $\mathrm{P}(E) = 1$, i.e. it will definitely occur.

7. If E is any event, then $\mathrm{P}(E') = 1 - \mathrm{P}(E)$, where $\mathrm{P}(E')$ is the probability that event E does not occur.

Mutually Exclusive Events

8. If events A and B cannot occur together, we say that they are mutually exclusive.

9. If A and B are mutually exclusive events, their sets of sample spaces are disjoint, i.e. $\mathrm{P}(A \cup B) = \mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B)$.

Example 3

A card is drawn at random from a standard pack of 52 playing cards. Find the probability of drawing a Queen or an ace.

Solution

Total number of possible outcomes = 52

P(drawing a Queen or an ace) = P(drawing a Queen) + P(drawing an ace)

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{2}{13}$$

Independent Events

10. If A and B are independent events, the occurrence of A does not affect that of B, i.e. $P(A \cap B) = P(A \text{ and } B) = P(A) \times P(B)$.

Possibility Diagrams and Tree Diagrams

11. Possibility diagrams and tree diagrams are useful in solving probability problems. The diagrams are used to list all possible outcomes of an experiment.

12. The sum of probabilities on the branches from the same point is 1.

Example 4

A red fair six-sided die and a blue fair six-sided die are rolled at the same time.

(a) Using a possibility diagram, show all the possible outcomes.

(b) Hence, find the probability that

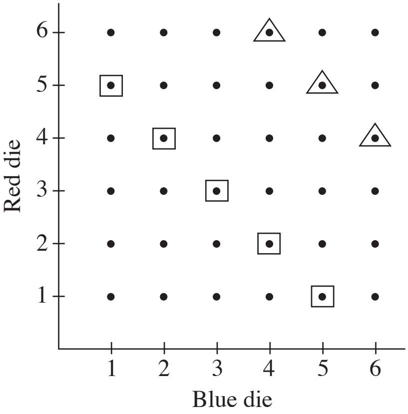
  (i) the sum of the numbers shown is 6,

  (ii) the sum of the numbers shown is 10,

  (iii) the red die shows a '3' and the blue die shows a '5'.

Solution

(a)



(b) Total number of possible outcomes = 6 × 6 = 36

  (i) There are 5 ways of obtaining a sum of 6, as shown by the squares on the diagram.

$$\therefore \text{P(sum of the numbers shown is 6)} = \frac{5}{36}$$

  (ii) There are 3 ways of obtaining a sum of 10, as shown by the triangles on the diagram.

$$\therefore \text{P(sum of the numbers shown is 10)} = \frac{3}{36}$$

$$= \frac{1}{12}$$

  (iii) P(red die shows a '3') = $\frac{1}{6}$

P(blue die shows a '5') = $\frac{1}{6}$

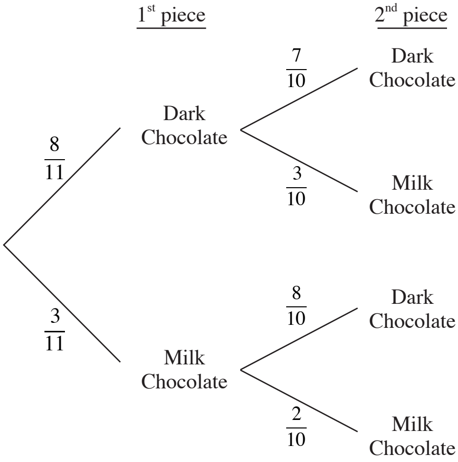
$$\text{P(red die shows a '3' and blue die shows a '5')} = \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{36}$$

Example 5

A box contains 8 pieces of dark chocolate and 3 pieces of milk chocolate. Two pieces of chocolate are taken from the box, without replacement. Find the probability that both pieces of chocolate are dark chocolate.

Solution



$$
\begin{aligned}
\text{P(both pieces of chocolate are dark chocolate)} &= \frac{8}{11} \times \frac{7}{10} \\
&= \frac{28}{55}
\end{aligned}
$$

Example 6

A box contains 20 similar marbles. 8 marbles are white and the remaining 12 marbles are red. A marble is picked out at random and not replaced. A second marble is then picked out at random.

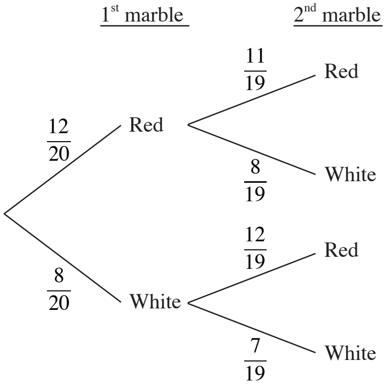
Calculate the probability that

(i) both marbles will be red,

(ii) there will be one red marble and one white marble.

Solution

Use a tree diagram to represent the possible outcomes.



(i) P(two red marbles) $= \frac{12}{20} \times \frac{11}{19}$

$= \frac{33}{95}$

(ii) P(one red marble and one white marble)

$= \left(\frac{12}{20} \times \frac{8}{19}\right) + \left(\frac{8}{20} \times \frac{12}{19}\right)$ (A red marble may be chosen first, followed by a white marble, and vice versa)

$= \frac{48}{95}$

Example 7

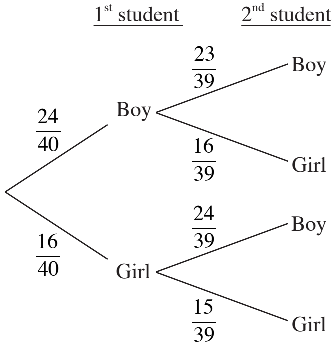
A class has 40 students. 24 are boys and the rest are girls. Two students were chosen at random from the class. Find the probability that

(i) both students chosen are boys,

(ii) a boy and a girl are chosen.

Solution

Use a tree diagram to represent the possible outcomes.



(i) P(both are boys) $= \frac{24}{40} \times \frac{23}{39}$

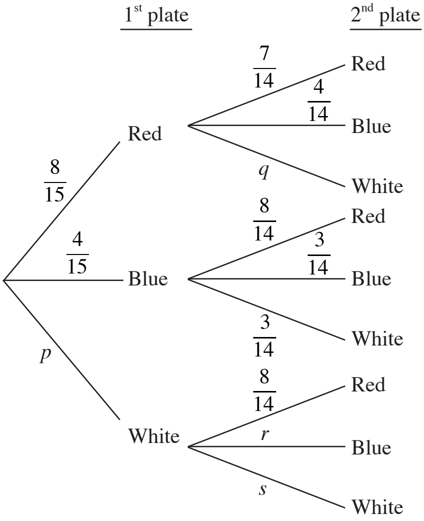
$= \frac{23}{65}$

(ii) P(one boy and one girl) $= \left(\frac{24}{40} \times \frac{16}{39}\right) + \left(\frac{16}{40} \times \frac{24}{39}\right)$ (A boy may be chosen first, followed by the girl, and vice versa)

$= \frac{32}{65}$

Example 8

A box contains 15 identical plates. There are 8 red, 4 blue and 3 white plates. A plate is selected at random and not replaced. A second plate is then selected at random and not replaced. The tree diagram shows the possible outcomes and some of their probabilities.



(a) Find the values of p, q, r and s.

(b) Expressing each of your answers as a fraction in its lowest terms, find the probability that

  (i) both plates are red,

  (ii) one plate is red and one plate is blue.

(c) A third plate is now selected at random. Find the probability that none of the three plates is white.

Solution

(a) $p = 1 - \frac{8}{15} - \frac{4}{15}$

$= \frac{1}{5}$

$q = 1 - \frac{7}{14} - \frac{4}{14}$

$= \frac{3}{14}$

$r = \frac{4}{14}$

$= \frac{2}{7}$

$s = 1 - \frac{8}{14} - \frac{2}{7}$

$= \frac{1}{7}$

(b) **(i)** P(both plates are red) $= \frac{8}{15} \times \frac{7}{14}$

$= \frac{4}{15}$

(ii) P(one plate is red and one plate is blue) $= \frac{8}{15} \times \frac{4}{14} + \frac{4}{15} \times \frac{8}{14}$

$= \frac{32}{105}$

(c) P(none of the three plates is white) $= \frac{8}{15} \times \frac{11}{14} \times \frac{10}{13} + \frac{4}{15} \times \frac{11}{14} \times \frac{10}{13}$

$= \frac{44}{91}$

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone } = \pi rl$$

$$\text{Surface area of a sphere } = 4\pi r^2$$

$$\text{Volume of a cone } = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere } = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

O Level Mathematics Topical Revision Notes is a comprehensive guide based on the latest syllabus. It is written to provide candidates sitting for the O Level and N(A) Level Mathematics examinations with thorough revision material. Important concepts and formulae are presented in simple and concise points for easier reference. Relevant worked examples are incorporated into the notes to facilitate the understanding of important concepts and formulae.

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